International Conference on Recent Innovation in Applied Science and Management
(IEI, Chandigarh) Institution of Engineers, India , Sector 19A, Chandigarh
12th October 2019 www.conferenceworld.in
ISBN : 978-81-941721-7-8

A PECULIAR CASE IN ASSIGNMENT PROBLEM WITH TRIANGULAR NUMBERS

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ABSTRACT

In this Article, we focus on an assignment Problem which contains Triangular numbers. It is considered by taking cost assignments as Triangular numbers. Some observations are traced. By using Hungarian Method, the optimum Assignment is found in the positions (1,n),(2,n-1),(3,n-2),(4,n-3)…….(n-3,4),(n-2,3),(n-1,2),(n,1) in the minimization type. The positions are reversed in the maximization type. By using Bottle neck Method, (1,1),(2,2),(3,3),(4,4)…….(n.n) are positions of optimum assignment in minimization and maximization types.

1. INTRODUCTION:

The triangular number sequence is 0,1,3,6,10,15,21,28,36,45….. It was developed by one of the great German mathematicians and eminent scientist Carl Friedrich Gauss in 1796 .The triangular numbers are obtained by n(n+1)/2 whire n=1,2,3,… In particular case, every positive integer is represented as a sum of triangular numbers He wrote in his dairy another number EYPHKA which has it's own significance. The triangular number does not satisfy the formula related to EYPHKA. Particularly Ramanjan's Theta Function is connected with sum of tri- angular numbers . Based on the Ramanujan-Nagell equation, $2^k - 1$ will be the form of largest triangular number i.e 4095. The handshake problem was also solved by the concept of Triangular numbers.

2. METHODOLOGY:

Hungarian method is one of the reputed methods for getting fine assignment to the given assignment problem.

Phases-(i): It is the first and foremost duty to check whether the given assignment problem is balanced or not. If it is not a balanced assignment problem, It should be converted as balanced assignment problem by introducing necessary dummy rows or dummy columns with zero cost values.

Phase(ii): In this phase, identify minimum element in the first row and subtract it from the remaining elements in the first row. The same process is adopted to the remaining all rows. In the same manner identify minimum element in the first column and subtract it only from the remaining elements in the first column. It is extended to the remaining all columns.

Phase(iii):By examining the rows consecutively to trace exactly the one zero in a row and make an assignment to the single zero by encircling it. The remaining other zeroes are crossed out in its column. Reiterate the same process till we reach the stage of having all zeroes of rows and columns are either encircled or crossed out.

ISBN : 978-81-941721-7-8

Phase(iv): Optimality condition stands on the criteria in which the number of assigned zeroes is equal to number of rows/columns. If the current phase is satisfying the optimality condition then the positions of assigned zeroes will supply the optimum assignment and the sum of the cost values of the respective positions will provide optimum solution (i.e.) Total minimum cost of the assignment problem.

Phase (v):At the stage of not satisfying optimality condition at any cycle, consider the concept of drawing minimum number of lines covering all zeroes including assigned zeroes and crossed out zeroes. It is always better to take the minimum number of lines less than or equal to the number of rows/columns

Phase (vi): Consider minimum element from the uncovered elements which will be subtracted from all the uncovered elements. This minimum element will be added at the position of intersection of horizontal and vertical lines. The remaining elements which are crossed by a single line remain unchanged. Repeat the process from phase (iv) until we reach the optimum solution.

The applications of some operations research techniques discussed by many mathematicians [1-18] are useful to trace a proper optimal solution by satisfying all the constraints.

3. BASIC ASSIGNMENT MODELS:

3.1 Case(i).:

The mathematical model of assignment problem in case (i) is defined as

$$Min / Max\ Z = \sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}x_{ij}$$

Subject to the constraints:

$\sum_{i=1}^{7} x_{ij} = 1$ for j=1,2,3,4,5,6 and 7 and $\sum_{j=1}^{7} x_{ij} = 1$ for i=1,2,3,4,5,6 and 7;

x_{ij} = either 0 or 1 for all i,j. Here x_{ij} denotes the assignment of i^{th} resource to j^{th} activity with the successive numbers of Triangular Numbers row wise.

Table-1- Tabular Form of 7x7 Assignment Problem with Triangular Number

7x7	1	II	III	IV	V	VI	VII
A	0	1	3	6	10	15	21
B	28	36	45	55	66	78	91
C	105	120	136	153	171	190	210
D	231	253	276	300	325	351	378
E	406	435	465	496	528	561	595
F	630	666	703	741	780	820	861
G	903	946	990	1035	1081	1128	1176

ISBN : 978-81-941721-7-8

Table-2: Hungarian Method With 7x7 Assignment Problems in Minimization Case

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization Type (7x7)	C1	(A,II), (B,I)	$P_{22},P_{23},P_{24},P_{25},P_{26},P_{27},P_{32},P_{33},P_{34},P_{35},P_{36},P_{37},P_{42},P_{43},P_{44},P_{45},P_{46},P_{47},P_{52},P_{53},P_{54},P_{55},P_{56},P_{57},P_{62},P_{63},P_{64},P_{65},P_{66},P_{67},P_{72},P_{73},P_{74},P_{75},P_{76},P_{77}$	2	(A,VII),(B,VI),(C,V),(D,IV),(E,III),(F,II),(G,I)	2604
	C2	(A,III),(B,II),(C,I)	$P_{32},P_{33},P_{34},P_{35},P_{36},P_{37},P_{42},P_{43},P_{44},P_{45},P_{46},P_{47},P_{52},P_{53},P_{54},P_{55},P_{56},P_{57},P_{62},P_{63},P_{64},P_{65},P_{66},P_{67},P_{72},P_{73},P_{74},P_{75},P_{76},P_{77}$	3		
	C3	(A,III),(B,II),(C,I)	$P_{33},P_{34},P_{35},P_{36},P_{37},P_{43},P_{44},P_{45},P_{46},P_{47},P_{53},P_{54},P_{55},P_{56},P_{57},P_{63},P_{64},P_{65},P_{66},P_{67},P_{73},P_{73},P_{74},P_{75},P_{76},P_{77}$	4		
	C4	(A,IV),(C,III),(D,I)	$P_{21},P_{22},P_{23},P_{24},P_{25},P_{26},P_{27}$	6		
	C5	(A,IV),(B,III),(C,II),(D,I)	$P_{42},P_{43},P_{44},P_{45},P_{46},P_{47},P_{52},P_{53},P_{54},P_{55},P_{56},P_{57},P_{62},P_{63},P_{64},P_{65},P_{66},P_{67},P_{72},P_{73},P_{74},P_{75},P_{76},P_{77}$	4		
	C6	(A,IV),(B,III),(C,II),(D,I)	$P_{43},P_{44},P_{45},P_{46},P_{47},P_{53},P_{54},P_{55},P_{56},P_{57},P_{63},P_{64},P_{65},P_{66},P_{67},P_{73},P_{74},P_{75},P_{76},P_{77}$	5		
	C7	(A,V),(B,IV),(C,II),(D,I)	$P_{22},P_{24},P_{25},P_{26},P_{27},P_{32},P_{34},P_{35},P_{36},P_{37},P_{52},P_{54},P_{55},P_{56},P_{57},P_{62},P_{64},P_{65},P_{66},P_{67},P_{72},P_{74},P_{75},P_{76},P_{77}$	4		
	C8	(A,V),(B,IV),(C,III),(D,II),(E,I)	$P_{53},P_{54},P_{55},P_{56},P_{57},P_{63},P_{64},P_{65},P_{66},P_{67},P_{73},P_{74},P_{75},P_{76},P_{77}$	6		
	C9	(A,VI),(B,V),(C,III),(D,II),(E,I)	$P_{41},P_{42},P_{43},P_{44},P_{45},P_{46},P_{47}$	6		
	C10	(A,VI),(B,V),(C,III),(D,II),(E,I)	$P_{32},P_{34},P_{35},P_{36},P_{37},P_{42},P_{44},P_{45},P_{46},P_{47},P_{62},P_{64},P_{65},P_{66},P_{67},P_{72},P_{74},P_{75},P_{76},P_{77}$	5		
	C11	(A,VI),(B,IV),(C,III),(D,II),(E,I)	$P_{44},P_{45},P_{46},P_{47},P_{54},P_{55},P_{56},P_{57},P_{64},P_{65},P_{66},P_{67},P_{74},P_{75},P_{76},$	6		

			P_{77}			
	C12	(A,VI),(B,V), (C,IV),(D,II), (E,I)	$P_{23},P_{25},P_{26},P_{27},P_{33},$ $P_{35},P_{36},P_{37},P_{63},P_{65},$ $P_{66},P_{67},P_{73},P_{75},P_{76},$ P_{77}	6		
	C13	(A,VII),(B,VI), (C,IV),(D,III), (E,II),(F,I)	$P_{51},P_{52},P_{53},P_{54},P_{55}$ $,P_{56},P_{57}$	6		
	C14	(A,VI),(B,V), (C,IV),(D,III), (E,II),(F,I)	$P_{63},P_{64},P_{65},P_{66},P_{67}$ $,P_{73},P_{74},P_{75},P_{76},P_{77}$	7		
	C15	(A,VII),(B,VI), (C,IV),(D,III), (E,II),(F,I)	$P_{41},P_{42},P_{44},P_{45},P_{46}$ $,P_{47},P_{51},P_{52},P_{54},P_{55}$ $,P_{56},P_{57}$	6		
	C16	(A,VI),(B,V), (C,IV),(D,III), (E,II),(F,I)	$P_{63},P_{64},P_{65},P_{66},P_{67},$ $P_{73},P_{74},P_{75},P_{76},P_{77}$	7		
	C17	(A,VII),(B,VI), (C,IV),(D,III), (E,II),(F,I)	$P_{41},P_{42},P_{45},P_{46},P_{47},$ $P_{51},P_{52},P_{55},P_{56},P_{57}$	7		
	C18	(A,VII),(B,VI), (C,V),(D,IV), (E,II),(F,I)	$P_{31},P_{32},P_{33},P_{34},P_{35},$ P_{36},P_{37}	6		
	C19	(A,VI),(B,V), (C,IV),(D,III), (E,II),(F,I)	$P_{63},P_{64},P_{65},P_{66},P_{67},$ $P_{73},P_{74},P_{75},P_{76},P_{77}$	7		
	C20	(A,VII),(B,VI), (C,IV),(D,III), (E,II),(F,I)	$P_{41},P_{42},P_{45},P_{46},P_{47},$ $P_{51},P_{52},P_{55},P_{56},P_{57}$	7		
	C21	(A,VII),(B,VI), (C,V),(D,IV), (E,II),(F,I)	$P_{21},P_{22},P_{23},P_{24},P_{26},$ $P_{27},P_{31},P_{32},P_{33},P_{34},$ P_{36},P_{37}	6		
	C22	(A,VII),(B,VI), (C,V),(D,IV), (E,III),(F,II), (G,I)	*	*		

Table-3: Hungarian Method With 7x7 Assignment Problem in Maximization

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
	C1	(A,VI),(B,VII)	$P_{11},P_{12},P_{13},P_{14},P_{15},$ $P_{16},P_{21},P_{22},P_{23},P_{24},$ $P_{25},P_{26},P_{31},P_{32},P_{33},$ $P_{34},P_{35},P_{36},P_{41},P_{42},$ $P_{43},P_{44},P_{45},P_{46},P_{51},$ $P_{52},P_{53},P_{54},P_{55},P_{56},$ $P_{61},P_{62},P_{63},P_{64},P_{65},$ $P_{66},P_{71},P_{72},P_{73},P_{74},$ P_{75},P_{76}	2		

	C2	(A,V),(B,VI), (C,VII)	P_{31},P_{32},P_{33},P_{34},P_{35}, P_{36},P_{41},P_{42},P_{43},P_{44}, P_{45},P_{46},P_{51},P_{52},P_{53}, P_{54},P_{55},P_{56},P_{61},P_{62}, P_{63},P_{64},P_{65},P_{66},P_{71}, P_{72},P_{73},P_{74},P_{75},P_{76}	3		
	C3	(A,V),(B,VI), (C,VII)	P_{31},P_{32},P_{33},P_{34},P_{35}, P_{41},P_{42},P_{43},P_{44},P_{45}, P_{51},P_{52},P_{53},P_{54},P_{55}, P_{61},P_{62},P_{63},P_{64},P_{65}, P_{71},P_{72},P_{73},P_{74},P_{75}	4		
	C4	(A,IV),(C,V), (B,VII)	P_{21},P_{22},P_{23},P_{24},P_{25}, P_{26},P_{27}	6		
	C5	(A,IV),(B,V), (C,VI),(D,VII)	P_{41},P_{42},P_{43},P_{44},P_{45}, P_{46},P_{51},P_{52},P_{53},P_{54}, P_{55},P_{56},P_{61},P_{62},P_{63}, P_{64},P_{65},P_{66},P_{71},P_{72}, P_{73},P_{74},P_{75},P_{76}	4		
	C6	(A,IV),(B,V), (C,VI),(D,VII)	P_{41},P_{42},P_{43},P_{44},P_{45}, P_{51},P_{52},P_{53},P_{54},P_{55}, P_{61},P_{62},P_{63},P_{64},P_{65} ,P_{71},P_{72},P_{73},P_{74},P_{75}	5		
	C7	(A,IV),(B,V), (D,VI),(E,VII)	P_{21},P_{22},P_{23},P_{24},P_{26}, P_{31},P_{32},P_{33},P_{34},P_{36}, P_{51},P_{52},P_{53},P_{54},P_{56}, P_{61},P_{62},P_{63},P_{64},P_{66}, P_{71},P_{72},P_{73},P_{74},P_{76}	4		
	C8	(A,III),(B,IV), (C,V),(D,VI), (E,VII)	P_{51},P_{52},P_{53},P_{54},P_{55}, P_{61},P_{62},P_{63},P_{64},P_{65} P_{71},P_{72},P_{73},P_{74},P_{75}	6		
	C9	*(A,III),(B,IV), (C,V),(E,VI), (F,VII)*	P_{41},P_{42},P_{43},P_{44},P_{45}, P_{46},P_{47}	6		
	C10	(A,III),(B,IV), (C,V),(E,VI), (F,VII)	P_{31},P_{32},P_{33},P_{34},P_{35}, P_{36},P_{41},P_{42},P_{43},P_{44}, P_{46},P_{61},P_{62},P_{63},P_{64}, P_{66},P_{71},P_{72},P_{73},P_{74} ,P_{76}	5		
	C11	(A,III),(B,IV), (C,V),(D,VII), (F,VII)	P_{41},P_{42},P_{43},P_{44},P_{51}, P_{52},P_{53},P_{54},P_{61},P_{62}, P_{63},P_{64},P_{71},P_{72},P_{73},P_{74}	6		
	C12	(A,III),(B,IV), (D,V),(E,VI), (F,VII)	P_{21},P_{22},P_{23},P_{25},P_{31}, P_{32},P_{33},P_{35},P_{61},P_{62}, P_{63},P_{65},P_{71},P_{72},P_{73},P_{75}	6		
	C13	(A,II),(B,III), (C,IV),(D,V), (F,VI),(G,VII)	P_{51},P_{52},P_{53},P_{54},P_{55} ,P_{56},P_{57}	6		
	C14	(A,II),(B,III), (C,IV),(D,V), (E,VI),(F,VII)	P_{61},P_{62},P_{63},P_{64},P_{65}, P_{71},P_{72},P_{73},P_{74},P_{75}	7		
Maximization Type (7x7)	C15	(A,II),(B,III), (C,IV),(D,V), (F,VI),(G,VII)	P_{41},P_{42},P_{43},P_{44},P_{46}, P_{47},P_{51},P_{52},P_{53},P_{54}, P_{56},P_{57}	6	(A,I),(B,II), (C,III),(D,IV), (E,V),(F,VI), (G,VII)	2996
	C16	(A,II),(B,III),	P_{61},P_{62},P_{63},P_{64},P_{65},	7		

		(C,IV),(D,V), (E,VI),(F,VII)	$P_{71},P_{72},P_{73},P_{74},P_{75}$			
	C17	(A,II),(B,III), (C,IV),(D,V), (F,VI),(G,VII)	$P_{41},P_{42},P_{43},P_{46},P_{47}$, $P_{51},P_{52},P_{53},P_{56},P_{57}$	7		
	C18	(A,II),(B,III), (D,IV),(E,V), (F,VI),(G,VII)	$P_{31},P_{32},P_{33},P_{34},P_{35}$, P_{36},P_{37}	6		
	C19	(A,II),(B,III), (C,IV),(D,V), (E,VI),(F,VII)	$P_{61},P_{62},P_{63},P_{64},P_{65}$, $P_{71},P_{72},P_{73},P_{74},P_{75}$	7		
	C20	(A,II),(B,III), (C,IV),(D,V), (F,VI),(G,VII)	$P_{41},P_{42},P_{43},P_{46},P_{47}$, $P_{51},P_{52},P_{53},P_{56},P_{57}$	7		
	C21	(A,II),(B,III), (D,IV),(E,V), (F,VI),(G,VII)	$P_{21},P_{22},P_{24},P_{25},P_{26}$, $P_{27},P_{31},P_{32},P_{34},P_{35}$, P_{36},P_{37}	6		
	C22	(A,I)(B,II)(C,III)(D,IV)(E,V)(F,VI)(G,VII)	*	*		

Table-4: Bottle Neck Method With 7x7 Assignment Problem in Minimization/ Maximization

Objective Function Type	Optimum Assignment	Total Assignment Cost
Minimization (7X7)	(A,VII),(B,VI),(C,V),(D,IV),(E,III),(F,II),(G,I)	2604
Maximization (7X7)	(A,VII),(B,VI),(C,V),(D,IV),(E,III),(F,II),(G,I)	2604

3.2 Case(ii).:

The mathematical model of assignment problem in case (ii) is defined as

$$Min / Max\ Z = \sum_{i=1}^{6}\sum_{j=1}^{6} c_{ij}x_{ij}$$

Subject to the constraints:

$\sum_{i=1}^{6} x_{ij} = 1$ for j=1,2,3,4,5and 6 and $\sum_{j=1}^{6} x_{ij} = 1$ for i=1,2,3,4,5and 6 ; x_{ij} = either 0 or 1 for all i,j

Here x_{ij} denotes the assignment of i^{th} resource to j^{th} activity with the successive numbers of Triangular Numbers row wise

ISBN : 978-81-941721-7-8

Table-5: Tabular Form of 6x6 Assignment Problem with Triangular Numbers

6x6	I	II	III	IV	V	VI
A	0	1	3	6	10	15
B	21	28	36	45	55	66
C	78	91	105	120	136	153
D	171	190	210	231	253	276
E	300	325	351	378	406	435
F	465	496	528	561	595	630

Table-6: Hungarian Method With 6x6 Assignment Problems in Minimization Case

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization Type (6x6)	C1	(A,II),(B,I)	$P_{22},P_{23},P_{24},P_{25},P_{26},P_{32},P_{33},P_{34},P_{35},P_{36},P_{42},P_{43},P_{44},P_{45},P_{46},P_{52},P_{53},P_{54},P_{55},P_{56},P_{62},P_{63},P_{64},P_{65},P_{66}$	2	(A,VI),(B,V),(C,IV),(D,III),(E,II),(F,I)	1190
	C2	(A,III),(B,II),(C,I)	$P_{32},P_{33},P_{34},P_{35},P_{36},P_{42},P_{43},P_{44},P_{45},P_{46},P_{52},P_{53}P_{54},P_{55},P_{56},P_{62},P_{63},P_{64},P_{65},P_{66}$	3		
	C3	(A,III),(B,II),(C,I)	$P_{33},P_{34},P_{35},P_{36},P_{43},P_{44},P_{45},P_{46},P_{53},P_{54},P_{55},P_{56},P_{63},P_{64},P_{65},P_{66},$	4		
	C4	(A,IV),(C,III),(D,I)	$P_{21},P_{22},P_{23},P_{24},P_{25},P_{26}$	5		
	C5	(A,IV),(B,III),(C,II),(D,I)	$P_{42},P_{43},P_{44},P_{45},P_{46},P_{52},P_{53},P_{54},P_{55},P_{56},P_{62},P_{63},P_{64},P_{65},P_{66}$	4		
	C6	(A,IV),(B,III),(C,II),(D,I)	$P_{43},P_{44},P_{45},P_{46},P_{53},P_{54},P_{55},P_{56},P_{63},P_{64},P_{65},P_{66}$	5		
	C7	(A,V),(B,IV),(C,II),(D,I)	$P_{22},P_{24},P_{25},P_{26},P_{32},P_{55},P_{56},P_{62},P_{64},P_{65},P_{66}$	4		
	C8	(A,V),(B,IV),(C,III),(D,II),(E,I)	$P_{53},P_{54},P_{55},P_{56},P_{63},P_{64},P_{65},P_{66}$	6		
	C9	(A,VI),(B,V),	$P_{41},P_{42},P_{43},P_{44},P_{45},$	5		

		(C,III),(D,II), (E,I)	P_{46}			
	C10	(A,II),(B,V), (C,III),(D,II), (E,I)	$P_{31},P_{32},P_{34},P_{35},P_{36}, P_{41},P_{42},P_{44},P_{45},P_{46}$	5		
	C11	(A,V),(B,IV), (C,III),(D,II), (E,I)	$P_{53},P_{54},P_{55},P_{56},P_{63}, P_{64},P_{65},P_{66}$	6		
	C12	(A,VI),(B,V), (C,III),(D,II), (E,I)	$P_{31},P_{32},P_{35},P_{36},P_{41}, P_{42},P_{45},P_{46}$	6		
	C13	(A,VI),(B,V),(C, IV),(D,III),(F,I)	$P_{21},P_{22},P_{23},P_{24},P_{25}, P_{26}$	5		
	C14	(A,VI),(B,V), (C,IV),(D,III), (E,II),(F,I)	*	*		

Table-7: Hungarian Method With 6x6 Assignment Problems in Maximization Case

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization Type (6x6)	C1	(A,V),(B,VI)	$P_{21},P_{23},P_{24},P_{25},P_{31}, P_{32},P_{33},P_{34},P_{35},P_{41}, P_{42},P_{43},P_{44},P_{45},P_{51}, P_{52},P_{53},P_{54},P_{55},P_{61}, P_{62},P_{63},P_{64},P_{65}$	2	(A,I),(B,II), (C,III),(D,IV), (E,V),(F,VI)	1400
	C2	(A,IV),(B,V), (C,VI)	$P_{31},P_{32},P_{33},P_{34},P_{35}, P_{41},P_{42},P_{43},P_{44},P_{45}, P_{51},P_{52},P_{53},P_{54},P_{55}, P_{61},P_{62},P_{63},P_{64},P_{65}$	3		
	C3	(A,III),(C,IV),(D, VI)	$P_{31},P_{32},P_{33},P_{34},P_{41}, P_{42},P_{43},P_{44},P_{51},P_{52}, P_{53},P_{54},P_{61},P_{62},P_{63}, P_{64}$	4		
	C4	(A,III),(B,IV), (C,V),(D,VI)	$P_{21},P_{22},P_{23},P_{24},P_{25}, P_{26}$	5		
	C5	(A,III),(B,IV), (C,V),(D,VI)	$P_{41},P_{42},P_{43},P_{44},P_{45}, P_{51},P_{52},P_{53},P_{54},P_{55}, P_{61},P_{62},P_{63},P_{64},P_{65}$	4		
	C6	(A,III),(B,IV), (D,V),(E,VI)	$P_{41},P_{42},P_{43},P_{44},P_{51}, P_{52},P_{53},P_{54},P_{61},P_{62}, P_{63},P_{64}$	5		
	C7	(A,II),(B,III), (C,IV),(D,V), (E,VI)	$P_{21},P_{22},P_{23},P_{25},P_{31}, P_{32},P_{33},P_{35},P_{51},P_{52}, P_{53},P_{55},P_{61},P_{62},P_{63}, P_{65}$	4		
	C8	(A,II),(B,III), (C,IV),(E,V), (F,VI)	$P_{51},P_{52},P_{53},P_{54},P_{61}, P_{62},P_{63},P_{64}$	6		
	C9	(A,II),(B,III), (C,IV),(E,V),	$P_{41},P_{42},P_{43},P_{44},P_{45}, P_{46},$	5		

		(F,VI)				
	C10	(A,II),(B,III), (C,IV),(D,V), (E,VI)	$P_{31},P_{32},P_{33},P_{35},P_{36},$ $P_{41},P_{42},P_{43},P_{45},P_{46}$	5		
	C11	(A,II),(B,III), (C,IV),(D,V), (E,VI)	$P_{51},P_{52},P_{53},P_{54},P_{61},$ P_{62},P_{63},P_{64}	6		
	C12	(A,II),(B,III), (C,IV),(E,V), (F,VI)	$P_{31},P_{32},P_{35},P_{36},P_{41},$ P_{42},P_{45},P_{46}	6		
	C13	(A,I),(C,III), (D,IV),(E,V), (F,VI)	$P_{21},P_{22},P_{23},P_{24},P_{25},$ P_{26}	5		
	C14	(A,I),(B,II), (C,III),(D,IV), (E,V),(F,VI)	*	*		

Table-8: Bottle Neck Method With 6x6 Assignment Problem in Minimization/Maximization

Objective Function Type	Optimum Assignment	Total Assignment Cost
Minimization	(A,VI),(B,V),(C,IV),(D,III), (E,II),(F,I)	1190
Maximization	(A,VI),(B,V),(C,IV),(D,III), (E,II),(F,I)	1190

3.3 Case(iii).:

The mathematical model of assignment problem in case (iii) is defined as

$$Min / Max\ Z = \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij}x_{ij}$$

Subject to the constraints:

$$\sum_{i=1}^{5} x_{ij} = 1 \text{ for j=1,2,3,4 and 5 and } \sum_{j=1}^{5} x_{ij} = 1 \text{ for i=1,2,3,4 and 5; } x_{ij} \text{ = either 0 or 1 for all i,j}$$

Here x_{ij} denotes the assignment of i^{th} resource to j^{th} activity with the successive numbers of Triangular Numbers row wise.

Table-9: Tabular Form of 5x5 Assignment Problem with Triangular Numbers

5x5	I	II	III	IV	V
A	0	1	3	6	10
B	15	21	28	26	45
C	55	66	78	91	105
D	120	136	153	171	190
E	210	231	253	276	300

ISBN : 978-81-941721-7-8

Table-10: Hungarian Method With 5x5 Assignment Problems in Minimization Case

Objective function type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization Type (5X5)	C1	(A,II),(B,I)	$P_{22},P_{23},P_{24},P_{25}P_{32},P_{33},P_{34},P_{35},P_{42},P_{43},P_{44},P_{45},P_{52},P_{53},P_{54},P_{55}$	2	(A,V), (B,IV), (C,III), (D,II), (E,I)	470
	C2	(A,III),(B,II), (C,I)	$P_{32},P_{33},P_{34},P_{35},P_{42},P_{43},P_{44},P_{45},P_{52},P_{53},P_{54},P_{55}$	3		
	C3	(A,III),(B,II), (C,I)	$P_{33},P_{34},P_{35},P_{43},P_{44},P_{45},P_{53},P_{54},P_{55}$	4		
	C4	(A,IV),(C,II), (D,I)	$P_{21},P_{22},P_{23},P_{24},P_{25}$	4		
	C5	(A,IV),(B,III), (C,II),(D,I)	$P_{42},P_{43},P_{44},P_{45},P_{52},P_{53},P_{54},P_{55}$	4		
	C6	(A,IV),(B,III), (C,II),(D,I)	$P_{43},P_{44},P_{45},P_{53},P_{54},P_{55}$	5		
	C7	(A,IV),(C,III), (D,II),(E,I)	$P_{21},P_{22},P_{24},P_{25},P_{31},P_{32},P_{34},P_{35}$	4		
	C8	(A,V),(B,IV), (C,III),(D,II), (E,I)	*	*		

Table :11 Hungarian Method With 5x5 Assignment Problems in maximization Case

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization (5x5)	C1	(A,IV),(B,V)	$P_{21},P_{22},P_{23},P_{24},P_{31},P_{32},P_{33},P_{34},P_{41},P_{42},P_{43},P_{44},P_{51},P_{52},P_{53},P_{54}$	2	(A,I),(B,II), (C,III), (D,IV),(E,V)	570
	C2	(A,III),(B,IV), (C,V)	$P_{31},P_{32},P_{33},P_{34},P_{41},P_{43},P_{44},P_{51},P_{52},P_{53},P_{54}$	3		
	C3	(A,III),(B,IV), (C,V)	$P_{31},P_{32},P_{33},P_{41},P_{42},P_{43},P_{51},P_{52},P_{53}$	4		
	C4	(A,I),(C,III), (D,V)	$P_{21},P_{22},P_{23},P_{24},P_{25}$	4		
	C5	(A,I),(BIII), (C,IV),(D,V)	$P_{41},P_{42},P_{43},P_{44},P_{51},P_{52},P_{53},P_{54}$	4		
	C6	(A,I),(B,III), (C,IV),(D,V)	$P_{41},P_{42},P_{43},P_{51}P_{52},P_{53}$	5		
	C7	(A,I),(B,III), (D,IV),(E,V)	$P_{21},P_{22},P_{24},P_{25},P_{31},P_{32},P_{34},P_{35}$	4		
	C8	(A,I),(B,II), (C,III),(D,IV), (E,V)	*	*		

ISBN : 978-81-941721-7-8

Table-12: Bottle Neck Method with 5x5 Assignment Problems in Minimization Case

Objective function type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization (5x5)	C1	(A,II),(B,I)	$P_{22},P_{32},P_{24},P_{25},P_{32},P_{33},P_{34},P_{35},P_{42},P_{43},P_{44},P_{45},P_{52},P_{53},P_{54},P_{55}$	2	(A,V), (B,IV), (C,III), (D,II), (E,I)	470
	C2	(A,II),(B,I)	$P_{32},P_{33},P_{34},P_{35},P_{42},P_{43},P_{44},P_{45},P_{52},P_{53},P_{54},P_{55}$	3		
	C3	(A,III),(B,II), (C,I)	$P_{33},P_{34},P_{35},P_{43},P_{44},P_{45},P_{53},P_{54},P_{55}$	4		
	C4	(A,IV),(C,III), (D,I)	$P_{21},P_{22},P_{23},P_{24},P_{25}$	4		
	C5	(A,IV),(B,III), (C,II),(D,I)	$P_{42},P_{43},P_{44},P_{45},P_{52},P_{53},P_{54},P_{55}$	4		
	C6	(A,IV),(B,III), (C,II),(D,I)	$P_{43},P_{44},P_{45},P_{53},P_{54},P_{55}$	5		
	C7	(A,IV),(B,III), (D,II),(E,I)	$P_{21},P_{22},P_{24},P_{25},P_{31},P_{32},P_{34},P_{35}$	4		
	C8	(A,V),(B,IV), (C,III),(D,II), (E,I)	*	*		

Table-13: Bottle Neck Method With 5x5 Assignment Problems in Maximization Case

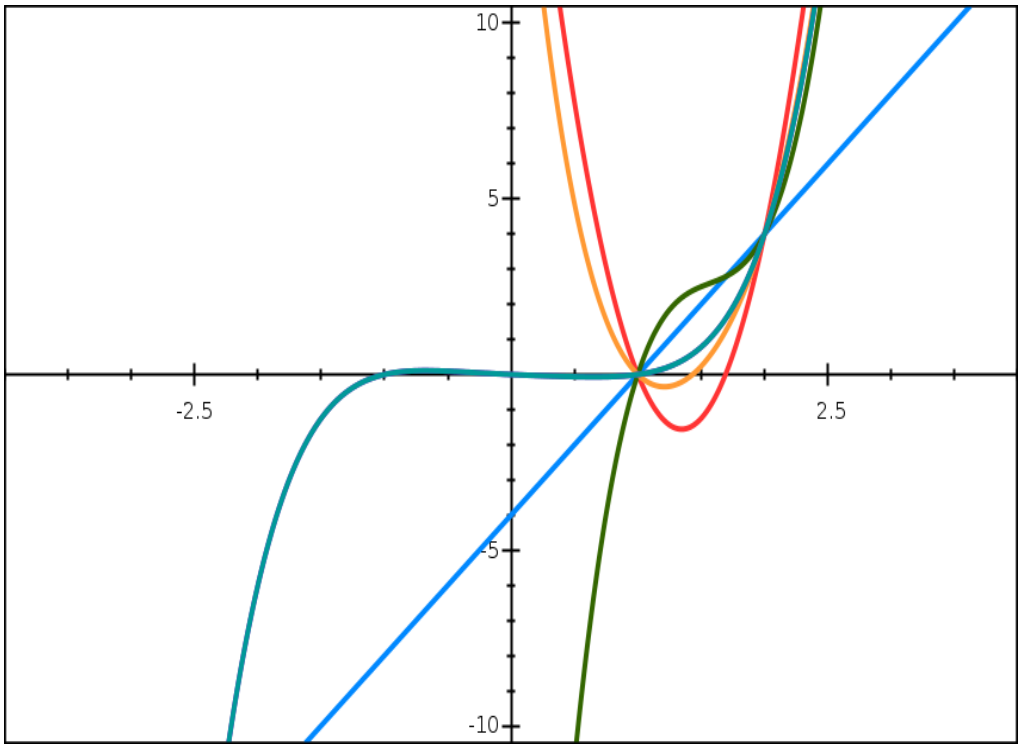
Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No.Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization (5x5)	C1	(D,IV),(E,V)	$P_{11},P_{12},P_{13},P_{14},P_{21},P_{22},P_{23},P_{24},P_{31},P_{32},P_{33},P_{34},P_{41},P_{42},P_{43},P_{44},$	2	(A,V), (B,IV), (C,III), (D,II), (E,I)	470
	C2	(C,V),(D,IV), (E,III)	$P_{11},P_{12},P_{13},P_{14},P_{21},P_{22},P_{23},P_{24},P_{31},P_{32},P_{33},P_{34}$	3		
	C3	(C,V),(D,IV), (E,III)	$P_{11},P_{12},P_{13},P_{21},P_{22},P_{23},P_{31},P_{32},P_{33}$	4		
	C4	(C,V),(D,III), (E,II)	$P_{41},P_{42},P_{43},P_{44},P_{45}$	4		
	C5	(D,V),(C,IV), (D,III),(E,II)	$P_{11},P_{12},P_{13},P_{14},P_{21},P_{22},P_{23},P_{24}$	4		
	C6	(B,V),(C,IV), (D,III),(E,II)	$P_{11},P_{12},P_{13},P_{21},P_{22},P_{23}$	5		
	C7	(B,V),(C,III), (D,II),(E,I)	$P_{31},P_{32},P_{34},P_{35},P_{41},P_{42},P_{44},P_{45}$	4		
	C8	(A,V),(B,IV), (C,III),(D,II), (E,I)	_	_		

ISBN : 978-81-941721-7-8

Based on the sizes of the assignment problems, the polynomials are derived and illustrated as below.

Table-14: Polynomials at different Cases

S.No	Cases	Polynomial
1	n=1,2	$4x-4$
2	n=1,2,3	$13x^2-35x+22$
3	n=1,2,3,4	$10x^3-47x^2+75x-38$
4	n=1,2,3,4,5	$2.416x^4-14.166x^3+37.583x^2-45.833x+20$
5	n=1,2,3,4,5,6	$0.166x^5-0.083x^4+0.083x^2-0.166x$
6	n=1,2,3,4,5,6,7	$0.16x^5-0.083x^4+0.083x^2-0.166x$



Grapgh-1

4. CONCLUSIONS:

(a).By using Hungarian Method,

(i).In the minimization type:the generalised optimum Assignment is in this model obtained in the positions (1,n),(2,n-1),(3,n-2),(4,n-3)…….(n-3,4),(n-2,3),(n-1,2),(n,1)

(ii).in the maximization type: the generalised optimum Assignment is occurred in the positions (1,n),(2,n-1),(3,n-2),(4,n-3)…….(n-3,4),(n-2,3),(n-1,2),(n,1)

(b).By using Bottle Neck Method:

(i).In the minimization type :the generalised optimum Assignment is gained in the positions (1,1),(2,2),(3,3),(4,4)…….(n.n)

ISBN : 978-81-941721-7-8

(ii).in the maximization type:the generalised optimum Assignment is existed in the positions (1,1),(2,2),(3,3),(4,4)…….(n.n)

(c) Uncovered elements are moved step by step in a constructive manner.

(iv).Exorbitant deviations are involved in various graphs of the polynomials.

5. REFERENCE:

[1].Alexander Schrijver, Theory of Linear and Integer Programming. John Wiley & Sons,1998

[2]. Billy E. Gillett, Introduction to operations Research, Tata McGraw-Hill Publishing Company limited, New York,1979.

[3]. Bland, Robert G. , "New Finite Pivoting Rules for the Simplex Method". Mathematics of Operations Research. 2 (2),1977, 103–107.

[4]. George B. Dantzig and Mukund N. Thapa.,Linear programming 1: Introduction. Springer-Verlag,1997.

[5].George B. Dantzig and Mukund N. Thapa., Linear Programming 2: Theory and Extensions. Springer-Verlag,2003.

[6]. S.D.Sharma, Operations Research, Kedar Nath Ram Nath & Co. ,1999.

[7].K.V.L.N.Acharyulu,A Special Case Study On 10x10 Symmetric Problem in Game Theory–Brown‟s Algorithm, Acta Ciencia Indica,Volume.43, No.2,2017,141-148.

[8].K.V.L.N.Acharyulu ,A Case Study On The Influence of Optimistic Time Estimate On A Network With Arithmetic Progression, International Journal of Advance research in science and Engineering (IJARSE),Volume.6, No.10 ,2017,1198-1205.

[9].K.V.L.N.Acharyulu, “Arithmetic Progression on Most Likely Time Estimate -A case Study”, Acta Ciencia Indica,Volume.43,No.2, 2017,165-172.

[10].K.V.L.N.Acharyulu , Prime Problem In Game Theory – Brown‟s Algorithm International Journal of Advance research in science and Engineering (IJARSE),Volume.6,No.10,2017,1206-1212.

[11].K.V.L.N.Acharyulu, “A Problem in Game Theory with Fibonacci Numbers” International Journal of Advance research in science and Engineering,Volume.6, No.11 ,PP.1954-1960,2017.

[12].K.V.L.N.Acharyulu ,A Game with non zeroTriangular numbers, Acta Ciencia Indica, Volume.43,No.4, 2017,247-253.

[13].K.V.L.N.Acharyulu, Pessimistic Time Estimate With Arithmetic Progression International Journal of Advance research in science and Engineering, Volume.6, No.11,2017,1961-1967.

[14].K.V.L.N.Acharyulu ,A Special symmetric Game Problem with Triangular Numbers- Brown‟s Algorithm, Acta Ciencia Indica,Volume.43,No.4,2017,221-227.

[15].Dr.K.V.L.N.Acharyulu, B.Sai Prasanna and B.Sri Satya Rajani,’A Special Case Study in LPP’,International Journal of Management, Technology and Engineering, Vol.8 ,Issue10,2018,1512-1521.

[16].Dr.K.V.L.N.Acharyulu, A.Bhargavi and G.Sravani, ‘A Peculiar Problem in Linear Programming Problem’, International Journal of Management, Technology and Engineering, Vol.8 ,Issue10,2018,1532-1540.

[17].Dr.K.V.L.N.Acharyulu, B.Jayasree and Sk .Mubeena, ‘A Generalized problem in Linear programming problem’, International Journal of Management, Technology and Engineering, Vol.8 ,Issue10,2018,1541-1548.

[18].Dr.K.V.L.N.Acharyulu, P.Hema and T.Vimala, ‘A Variety Problem in Linear Programming’, International Journal of Management, Technology And Engineering, Vol.8 ,Issue10,2018,1549-1556.