Radiation due to Rotational Motion by a Submerged Cylindrical Structure in Water

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ABSTRACT

In this paper, we study hydrodynamic coefficients because of rotational movement of submerged structure is extraordinary huge to planning a device which can be consider as a device of wave vitality. In view of a hypothetical methodology, we created to portray radiation of water wave by completely submerged cylinder set over a submerged roundabout plate in water of limited profundity which depends on linear water wave hypothesis The radiation problem because of rotational movement by this pair of cylinders have researched with the doubt of linear water wave hypothesis. To determine the radiated potentials in every region, we used the eigenfunction expansion method and variables separation method. From that point onward, we inferred the analytical expression of Hydrodynamic coefficients I. e. added mass and damping coefficient because of rotational motion and associated unknown coefficients are calculated by utilizing the matching conditions between the physical and virtual boundaries. A set of added mass and damping coefficient have presented graphically for various radius of the lower submerge cylinder and draft of the upper cylinder.

KEYWORDS: **Finite, virtual, added mass, damping coefficient, radiation.**

I. INTRODUCTION

In the study of radiation due to floating structures provide us useful properties such as hydrodynamic property. Water wave radiating problem involving regular structures in water of finite depth are yet to be investigated by many researchers. In our study we have consider the propagating of water wave on a pair of coaxial cylinder with the suspicion of linear water wave theory.

In [2] and [7] discussed the interaction of waves with a cylinder in water of uniform depth and determined the analytical expression of velocity potential for both interior and exterior regions. In [3] calculated wave induced force acting on the floating rectangular structure placed at near a wall in water of step type base. In [5-6] discussed water wave forces due to a device consist of two cylinders which placed in water of uniform depth. In [8] gave an analytical expression of diffracted velocity potential for a single vertical cylinder in water of arbitrary depth. In [10] developed non-linear water wave theory and they investigated second order force for a couple of cylinders using Graft's addition theorem. In [11] analysed the scattering and radiation of water wave due to a rectangular oscillating structure considering a bottom still effect. In [12-13] analysed the scattering and radiation of water waves by couple of cylinders which are placed in water of constant depth. Also in [14] formulated the problem of scattering and radiation by considering a couple of vertical truncated cylinders in water of constant depth. In [4] discussed the radiation problem due to surge motion by pair of submerged cylindrical structure and they derived hydrodynamic coefficient as well as plot graphically.

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II. FORMULATION

Based on the hypothesis of linear water wave propagation in ideal fluid of finite depth H with oscillating submerged cylinder placed above a circular plat which is fixed at finite height from the sea bottom. Assume the radius of submerged cylinder is R that occupies the region $r \le R, 0 \le \theta \le 2\pi, -l_3 \le z \le -l_4$ and the radius of lower cylinder is $R_b (\ge R)$ that occupies the region $r \le R_b, 0 \le \theta \le 2\pi, -l_1 \le z \le -l_2$. Also the Cartesian coordinate chose origin at $O,$ as shown in Fig. 1 in which $z-$ axis is measured vertically upward.

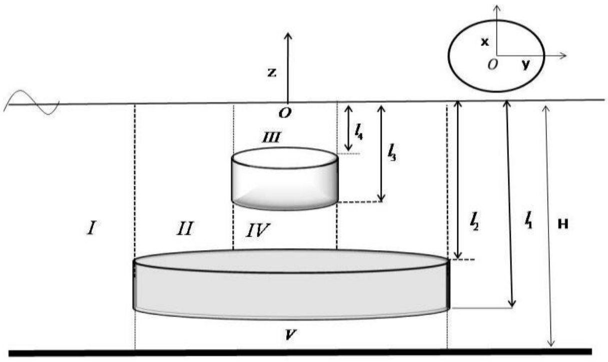

Fig. 1. Device

Bases on linear water wave theory, we have the velocity potential of the form as

$$\phi(r,\theta,z,t) = \text{Re}[\phi(r,\theta,z)e^{-i\omega t}], \tag{1}$$

where Re[.] represents the real part of a complex number, $i = \sqrt{-1}$ and $\varphi(r,\theta,z)$ is the time-independent velocity potential which must satisfies the Laplace's equation, i. e.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial \phi}{\partial z^2} = 0. \tag{2}$$

A. Governing equation and boundary conditions

The radiated potential ϕ_r for the roll motion is given by (as in [10])

$$\phi_r = -i\omega\varphi_{r2}(r,z)\cos\theta \tag{3}$$

Therefore, we have

$$\frac{1}{r}\frac{\partial \varphi_{r2}}{\partial r} + \frac{\partial^2 \varphi_{r2}}{\partial r^2} + \frac{\partial \varphi_{r2}}{\partial z^2} - \frac{\varphi_{r2}}{r^2} = 0, \tag{4}$$

$$\frac{\partial \varphi_{r2}}{\partial z} - \frac{\omega^2}{g}\varphi_{r2} = 0 \quad (z = 0) \qquad (5)$$

$$\frac{\partial \varphi_{r2}}{\partial z} = 0 \quad (z = -H) \qquad (6)$$

$$\frac{\partial \varphi_{r2}}{\partial z} = 0 \quad (z = -l_2, r < R_b; z = -l_4, r < R) \qquad (7)$$

$$\frac{\partial \varphi_{r2}}{\partial r} = \begin{cases} 0, & -l_1 < z < -l_2, r = R_b \\ (z - z'), & -l_3 < z < -l_4, r = R \end{cases} \qquad (8)$$

$$\lim_{r\to\infty} \sqrt{kr}\left(\frac{\partial \varphi_{r2}}{\partial r} - ik\varphi_{r2}\right) = 0, \qquad (9)$$

where $(0,0,z')$ is the centre of rotation. The fluid regions as indicated in Fig. 1, namely I, II, III, IV and V. Let us consider the radiated velocity potentials are $\varphi_{r2}{}^{I}, \varphi_{r2}{}^{II}, \varphi_{r2}{}^{III}, \varphi_{r2}{}^{IV}$ and $\varphi_{r2}{}^{V}$ in the respected regions,.

B. Matching conditions

Along the boundary $r = R_b$ to preserve continuity, we have

$$\varphi_{r2}^{I} = \varphi_{r2}^{II} \quad (-l_2 \le z \le 0) \qquad (10)$$

$$\varphi_{r2}^{I} = \varphi_{r2}^{V} \quad (-H \le z \le -l_1) \qquad (11)$$

$$\frac{\partial \varphi_{r2}^{I}}{\partial r} = \begin{cases} \frac{\partial \varphi_{r2}^{II}}{\partial r} & (-l_2 \le z \le 0) \\ 0 & (-l_1 \le z \le -l_2) \\ \frac{\partial \varphi_{r2}^{V}}{\partial r} & (-H \le z \le -l_1) \end{cases} \qquad (12)$$

Along the boundary $r = R$, we have

$$\varphi_{r2}^{II} = \begin{cases} \varphi_{r2}^{III} & (-l_4 \le z \le 0) \\ \varphi_{r2}^{IV} & (-l_2 \le z \le -l_3) \end{cases} \qquad (13)$$

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$$\frac{\partial \varphi_{r2}^{II}}{\partial r} = \begin{cases} \frac{\partial \varphi_{r2}^{III}}{\partial r} & (-l_4 \le z \le 0) \\ (z - z') & (-l_3 \le z \le -l_4) \\ \frac{\partial \varphi_{r2}^{IV}}{\partial r} & (-l_2 \le z \le -l_3) \end{cases} \tag{14}$$

C. Radiated potentials and hydrodynamic coefficients

Solution of above boundary value problem based on the method of variables separatio. Therefore the radiated potential for each region is given by

$$\varphi_{r2}^{I} = \sum_{n=0}^{\infty} A_n \frac{U_1(\mu_n r)}{U_1(\mu_n R_b)} \cos[\mu_n (z+H)], \tag{15}$$

$$\varphi_{r2}^{II} = \sum_{n=0}^{\infty} \left[B_n \frac{V_1(\nu_n r)}{V_1(\nu_n R)} + C_n \frac{W_1(\nu_n r)}{W_1(\nu_n R)} \right] \cos[\nu_n (z+l_2)] \tag{16}$$

$$\varphi_{r2}^{III} = \sum_{n=0}^{\infty} D_n \frac{R_1(\gamma_n r)}{R_1(\gamma_n R)} \cos[\gamma_n (z+l_4)], \tag{17}$$

$$\varphi_{r2}^{IV} = \left[E_0 r + \sum_{n=1}^{\infty} E_n \frac{I_1(\delta_n r)}{I_1(\delta_n R)} \cos[\delta_n (z+l_2)] \right], \tag{18}$$

$$\varphi_{r2}^{V} = F_0 r + \sum_{n=1}^{\infty} F_n \frac{I_1(\lambda_n r)}{I_1(\lambda_n R_b)} \cos[\lambda_n (z+H)] \tag{19}$$

where A_n, B_n, C_n, D_n, E_n and F_n are the unknown constants and $\mu_n, \nu_n, \gamma_n, \delta_n$ and λ_n are the eigen values which are calculated by using the following dispersion relation:

$$\begin{cases} \mu_n = -ik & \omega^2 = gk \tanh(kH), n = 0 \\ \omega^2 = -g\mu_n \tan(\mu_n H) & n = 1, 2, \ldots \end{cases} \tag{20}$$

$$\begin{cases} \nu_n = -ik_2 & \omega^2 = gk_2 \tanh(k_2 l_2), n = 0 \\ \omega^2 = -g\nu_n \tan(\nu_n l_2) & n = 1, 2, \ldots \end{cases} \tag{21}$$

$$\begin{cases} \gamma_n = -ik_3 & \omega^2 = gk_3 \tanh(k_3 l_4), n = 0 \\ \omega^2 = -g\gamma_n \tan(\gamma_n l_4) & n = 1, 2, \ldots \end{cases} \tag{22}$$

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$$\delta_n = \frac{n\pi}{l_2 - l_3} \qquad n = 0, 1, 2, \ldots \tag{23}$$

$$\lambda_n = \frac{n\pi}{H - l_1} \qquad n = 0, 1, 2, \ldots \tag{24}$$

where k, k_2 and k_3 are the wave numbers in respected regions I , II and III .

The Parameters $U_1(.), V_1(.), W_1(.)$ and $R_1(.)$ are given by

$$U_1(\mu_n r) = H_1^{(1)}(kr) = H_1^{(1)}(i\mu_0 r), \quad n = 0 \tag{25}$$

$$U_1(\mu_n r) = K_1(\mu_n r), \quad n = 1, 2, \ldots \tag{26}$$

$$V_1(\nu_n r) = H_1^{(1)}(k_2 r), \quad n = 0 \tag{27}$$

$$V_1(\nu_n r) = K_1(\nu_n r), \quad n = 1, 2, \ldots \tag{28}$$

$$W_1(\nu_n r) = H_1^{(2)}(k_2 r), \quad n = 0 \tag{29}$$

$$W_1(\nu_n r) = I_1(\nu_n r), \quad n = 1, 2, \ldots \tag{30}$$

$$R_1(\gamma_n r) = J_1(k_3 r), \quad n = 0 \tag{31}$$

$$R_1(\gamma_n r) = I_1(\gamma_n r), \quad n = 1, 2, \ldots \tag{32}$$

Hence the hydrodynamic coefficients due to rotation motion of the cylinder are derived and hydrodynamic coefficients are called added mass and damping coefficient. These are given as follows:

$$F_r = \mu_2 + i\frac{\xi_2}{\omega}, \tag{33}$$

where, μ_2 is called added mass and ξ_2 is called damping coefficient due to surge motion. Therefore, applying equations , we get the expression of added mass and damping coefficient and these are given by

$$\mu_2 + i\frac{\xi_2}{\omega} = -\rho \iint_W \varphi_{r2}^{II}(R,z)\cos\theta(z - z')n_x ds, \tag{34}$$

$$\mu_2 + i\frac{\xi_2}{\omega} = -\pi\rho R \sum_{n=0}^{\infty}(B_n + C_n)\int_{-l_3}^{-l_4}\cos[\beta_n(z + l_2)].(z - z')dz. \tag{35}$$

III. METHOD TO DETERMINE THE UNKNOWN CONSTANTS

The unknown constant to be determine by applying the above matching conditions. Therefore we get

$$\int_{-l_2}^{0} \varphi_{r2}^{I}(R_b,\theta,z).\cos[\nu_l(z+l_3)]\,dz = \int_{-l_2}^{0} \varphi_{r2}^{II}(R_b,\theta,z).\cos[\nu_l(z+l_3)]\,dz \qquad (34)$$

$$\int_{-H}^{-l_1} \varphi_{r2}^{I}(R_b,\theta,z).\cos[\delta_l(z+H)]\,dz = \int_{-H}^{-l_1} \varphi_{r2}^{V}(R_b,\theta,z).\cos[\delta_l(z+H)]\,dz \qquad (34)$$

$$\int_{-H}^{0} \frac{\partial\varphi_{r2}^{I}(R_b,\theta,z)}{\partial r}.\cos[\mu_m(z+H)]\,dz = \int_{-l_2}^{0} \frac{\partial\varphi_{r2}^{II}(R_b,\theta,z)}{\partial r}.\cos[\mu_m(z+H)]\,dz + \int_{-H}^{-l_1} \frac{\partial\varphi_{r2}^{V}(R_b,\theta,z)}{\partial r}.\cos[\mu_m(z+H)]\,dz$$

$$(35) \quad \int_{-l_4}^{0} \varphi_{r2}^{II}(R,\theta,z).\cos[\gamma_m(z+l_4)]\,dz = \int_{-l_4}^{0} \varphi_{r2}^{III}(R,\theta,z).\cos[\gamma_m(z+l_4)]\,dz$$

$$(36) \quad \int_{-l_4}^{0} \varphi_{r2}^{II}(R,\theta,z).\cos[\gamma_m(z+l_4)]\,dz = \int_{-l_4}^{0} \varphi_{r2}^{III}(R,\theta,z).\cos[\gamma_m(z+l_4)]\,dz$$

$$(37)$$

$$\int_{-l_2}^{0} \frac{\partial\varphi_{r2}^{II}(R,\theta,z)}{\partial r}.\cos[\mu_m(z+l_2)]\,dz = \int_{-l_2}^{-l_3} \frac{\partial\varphi_{r2}^{III}(R,\theta,z)}{\partial r}.\cos[\mu_m(z+l_2)]\,dz + \int_{-l_3}^{-l_4} (z-z').\cos[\mu_m(z+l_2)\,dz + \int_{-l_4}^{0} \frac{\partial\varphi_{r2}^{IV}(R,\theta,z)}{\partial r}.\cos[\mu_m(z+l_2)\,dz \qquad (38)$$

Again, we assume

$$N(x_n, a_1, h_1, h_2) = \int_{h_1}^{h_2} \cos^2[x_n(z+a_1)]\,dz.$$

$$(39) \quad M(x_n, y_n, a_1, a_2, h_1, h_2) = \int_{h_1}^{h_2} \cos[x_n(z+a_1)].\cos[y_n(z+a_2)]\,dz, \qquad (40)$$

Applying equations (39) and (40) to equations (33)−(38), we get

$$\sum_{n=0}^{\infty} A_n M(\mu_n, \nu_m, H, l_2, -l_2, 0) = \left[B_m S_m^{'} + C_m T_m^{'}\right] N(\nu_m, l_2, -l_2, 0) \qquad (41)$$

$$A_m N(\mu_m, H, -H, 0) = \sum_{n=0}^{\infty} [B_n Q_n + C_n S_n] M(\mu_m, \nu_n, H, l_2, -l_2, 0) \qquad (42)$$

$$\sum_{n=0}^{\infty} (B_n + C_n) M(\nu_n, \gamma_m, l_2, l_4, -l_4, 0) = D_m N(\gamma_m, l_4, -l_4, 0) \qquad (43)$$

$$\sum_{n=0}^{\infty} (B_n + C_n) M(\nu_n, \delta_m, l_2, l_2, -l_2, -l_3) = E_m L_m N(\delta_m, l_2, -l_2, -l_3) \qquad (44)$$

$$[B_m O_m + C_m T_m] N(\nu_m, l_2, -l_2, 0) = \sum_{n=0}^{\infty} D_n X_n M(\gamma_n, \nu_m, l_2, l_4, -l_4, 0) + \int_{-l_3}^{-l_4} (z - z').\cos[\nu_m (z + l_2) dz + \sum_{n=0}^{\infty} E_n X_n M(\delta_n, \nu_m, l_2, l_2, -l_2, -l_3) \quad (45)$$

where

$$S_m^{'} = \frac{V_1(\nu_m R_b)}{V_1(\nu_m R)},\; T_m^{'} = \frac{W_1(\nu_m R_b)}{W_1(\nu_m R)}.$$

$$P_m = \frac{\varsigma_m U_1^{'}(\mu_m R_b)}{U_1(\mu_m R)},\; Q_n = \frac{\nu_n V_1^{'}(\nu_n R_b)}{V_1(\nu_n R)}$$

$$S_n = \frac{\nu_n W_1^{'}(\nu_n R_b)}{W_1(\nu_n R)},\quad L_m = \begin{cases} R & m = 0 \\ 1 & m = 1, 2, ... \end{cases}$$

$$O_m = \frac{\nu_m V_1^{'}(\nu_m R)}{V_1(\nu_m R)}, T_m = \frac{\nu_m W_1^{'}(\nu_m R)}{W_1(\nu_m R)},\; X_n = \begin{cases} 1 & n = 0 \\ \dfrac{\nu_n I_1^{'}(\nu_n R)}{I_1(\nu_n R)} & n = 1, 2, ... \end{cases}$$

IV. RESULT AND CONCLUSION

Let us consider the values of the parameters throughout our calculation as $H = 3m,\, g = 9.8m/s^2,\, l_2 = 0.75m$. In Figs. (2) and (3), respectively, we plot non dimensional added mass and damping coefficient versus dimensionless wave number for different values of radii of lower cylinder by taking $R_b = 1R, 2R, 3R$. Here we fixed the values of draft of the cylinders. From Fig. (2), it is clear that the added mass is enduring and positive for the lower range of wave number k, but at particular value of wave number (also at a particular frequency as there is a relation between wave number and frequency which is given by dispersion relation) at $kR = 2.4$ (approximately near), the behaviour of added mass very highly oscillate which we can take as a resonant situation. Fig (3) shows that the value of damping coefficient oscillated for lower range of wave number and for higher value of R_b, the value of damping coefficient gives maximum, for example $R_b = 3R$.

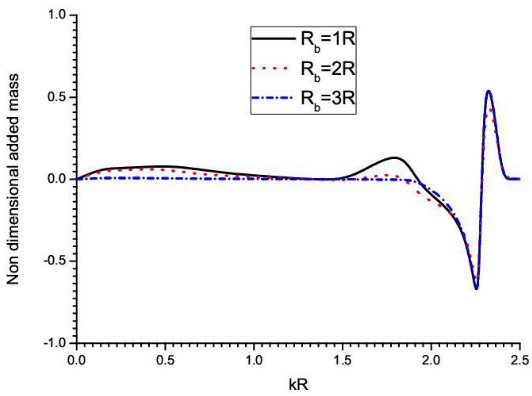

Fig. 2:Effect of different radii R_b on added mass

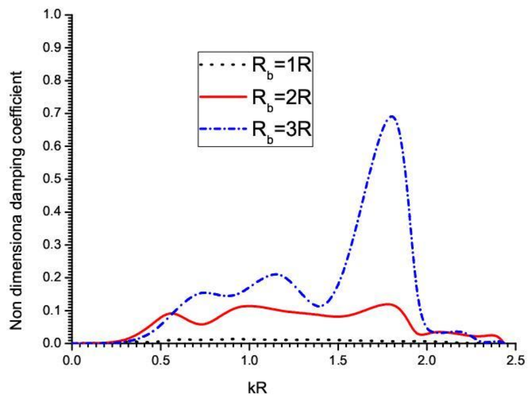

Fig.3:Effect of different radii R_b on damping coefficient

Again in Figs. (4) and (5), respectively, we plot non dimensional added mass and damping coefficient versus dimensionless wave number for different draft of the upper cylinder i. e. $l_3 = 0.15l_2, 0.20l_2, 0.25l_2$ and here we fixed the radius of lower cylinder by taking $R_b = 2R$. From Fig. (4) observed that same kind of behaviour as in Fig. (2), i.e. we have a resonant situation at particular wave number for all values of R_b. Smilarly, effect of draft on the damping coefficient shown in Fig. (5). From figure we observed that the value of damping coefficient oscillated for lower range of wave number and for higher value of R_b, the value of damping coefficient gives maximum, for example $R_b = 3R$.

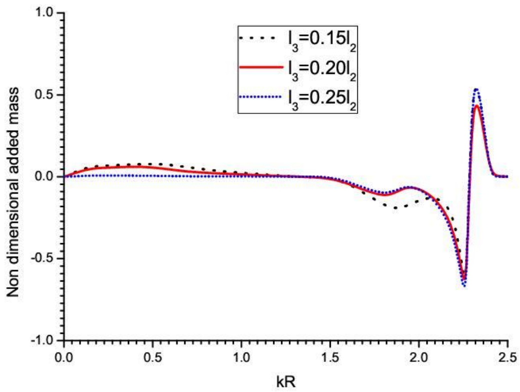

Fig. 4:Effect of different draft of the upper cylinder on added mass.

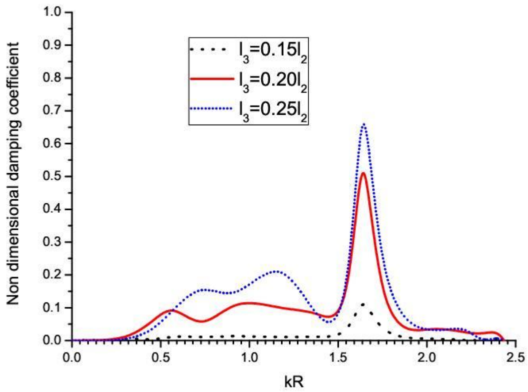

Fig.5: Effect of different draft of the upper cylinder on damping coeeficients.

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