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Amazing facts about NEAR-RINGS

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ABSTRACT:

In this paper, we deal with the amazing facts about near-rings , more exciting examples of near – rings and the derivation in near-rings.

Keywords : Near-ring, prime near-rings , commutative near-rings, derivation in Near-rings.

I.INTRODUCTION

The first ones to use the name near-ring were Zassenhauss, Blackett and P.Jordan. The late fifties of the last century brought the start of a rapid development of the theory of near-rings.The purpose of this paper is to study the derivation of near-rings.

Throughout this paper,N always stands for Near-rings, Z for the center and d for the derivation of the near-rings

Definition.1.1 ***(left near-ring /right near- ring).*** Let N be a non empty set equipped with two binary operations say '+' and '.'. N is called a left near-ring (resp. right near-ring) if

(i) (N,+) is a group (not necessarily abelian).

(ii) (N, .) is a semigroup.

(iii) $x.(y+z) = x.y + x.z$ for all $x,y,z \in N$ (resp. $(y+z).x = y.x + x.z$ for all $x,y,z \in N$)

<u>Some amazing examples of Near-rings</u>

1. Every ring is also a near-ring. (*see figure 1*)

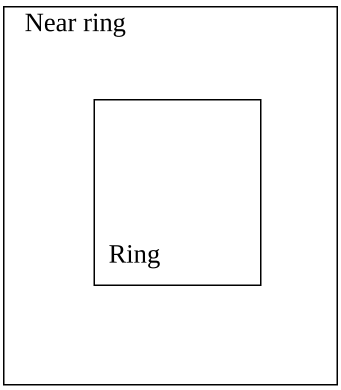

Figure 1

2. The mapping of a group into itself under pointwise addition and composition of maps is a Near -ring.
3. (R[x] , + , o) where R is a commutative ring with identity is a near-ring.
4. Let (C , +) be usual group of complex numbers with regard to ordinary addition of complex numbers. Let us define $'*'$ in C as following

$$a * b = |a|.b \quad \text{for all } a,b \in C$$

Then (C,+, $*$) is a left near-ring which is not a right near-ring.

Definition.1.2. A left near-ring N is called *zero Symmetric* if

$$0x = 0 \text{ for all } x \in N$$

Definition.1.3 A left near-ring N is called prime near-ring if

$xNy = \{0\}$ where $x,y \in N$

implies $x = 0$ or $y = 0$.

It is called Semi prime near-ring if

$$xNx = 0 \quad \text{where } x \in N \text{ implies } x = 0.$$

Definition.1.4. Let N be a near-ring. Then N is called a distributive

near-ring if

$$(y + z) x = yx + zx \qquad \text{for all } x,y,z \in N$$

Definition 1.5. Let N be a near-ring. Then N is called a commutative

near-ring if

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xy = yx for all x,y $\in$ N

Definition 1.6. The multiplicative center of near-ring N,usually denoted by Z is defined as;

Z = { x $\in$ N | xy = yx for all y $\in$ N }.

The additive center of N is defined as { x $\in$ N | x +y = y + x for all y $\in$N }

Definition.1.7. Let N be a near-ring and K a non empty subset of N.then a normal subgroup $(K,+)of\ (N,+)is\ called\ a\ left\ ideal\ (resp. a\ right\ ideal\ )of\ N\ \ if$

$xk \in K$

(resp. (x+k)y – xy $\in$ K) holds for all x,y $\in$ N and for all k $\in$ K. K is called an ideal of N.if it is both a left ideal as well as a right ideal of N.

Derivation in near-rings

The notion of derivation in rings is quite old and plays a significant role in the in integration analysis, algebraic geometry and algebra.The concepts of derivation, symmetric bi-derivation,permuting tri-derivation and permuting derivation have already been introduced in rings by G.Maksa, M.A.Ozturk and K.H.Park, H.E.Bell [1].In this paper ,we define all the derivation in near rings.

Definition 1.8. A derivation d on N is defined to be an additive endomorphism satisfying the product rule d(xy) = xd(y) + d(x)y for all x,y $\in$ N

Example. Let us consider (C,+,*) where * is defined as x * y = $|x|.y$

for all x,y $\in$ C, then it can be easily seen that (C,+,*) is a zero symmetric left near-ring which is not a right near-ring.

Assume N = $\left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in C \right\}$,then N is a zero symmetric left near-ring

which is not a right near-ring.Define d : N $\rightarrow$ N as following

d$\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$. Then d is non zero derivation on N.

A near-ring is not necessarily commutative; the following result1 has its own significance.

Result 1: Let d be an arbitrary additive endomorphism of N .then

d(xy) = xd(y) + d(x)y for all x,y ∈ N if and only if d(xy) = d(x)y +

xd(y). Therefore d is a derivation iff

d(xy) = d(x)y + xd(y) .

Proof. Suppose d(xy) = xd(y) + d(x)y for all x,y ∈ N.

Since x(y + y) = xy + xy and

d(x(y+y)) = xd(y + y) + d(x)(y + y) = xd(y) + xd(y) + d(x)y + d(x)y

also d(xy + xy) = d(xy) + d(xy) = xd(y) + d(x)y + xd(y) + d(x)y,

we get xd(y) + d(x)y = d(x)y + xd(y)

so d(xy) = d(x)y + xd(y)

The converse is proved in a similar way.

Definition 1.9. (***right generalized derivation of a near-ring***). Let N be a near-ring . then an additive mapping f : N $\rightarrow$ N is called a *right* *generalized derivation* of N if there exists a derivation d of N such that

f(xy) = f(x)y + xd(y) for all x,y $\in$ N.

Definition 2.0. (***left generalized derivation of a near-ring).*** Let N be a near-ring . then an additive mapping $f : N \to N$ is called a *left generalized derivation* of N if there exists a derivation d of N such that

f(xy) = d(x)y + xf(y) for all x,y $\in$ N.

Definition 2.1.(*generalized derivation of near-ring).* Let N be a near-ring. then an additive mapping $f : N \to N$ is called a generalized derivation of N if it is left as well as right generalized derivation of N.

Example. Let N be any zero symmetric left near-ring. Consider

$N_1 = \left\{ \begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} | \, 0, a, b \in N \right\}$ Then N_1 is a zero symmetric left near-ring with regard

To the matrix addition and multiplication.Define d: $N_1 \to N_1$ and

f: $N_1 \to N_1$ as d$\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$ and f$\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix}$. It can be easily seen

that f is a right generalized derivation of N_1 with associated derivation d of N_1 but it is not a left generalized derivation of N_1 with associated derivation of N_1.

REFERENCES

1. H. E. Bell and G. Mason, On derivations in near-ring, Near-Rings and Near-Fields (G.Betsch, ed.), North-Holland, Amsterdam, 1987, pp. 31-35.