Thermo-mechanics of magneto-micropolar composite

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Abstract: The purpose of this paper is to study the thermo-mechanics of magneto-micropolarthermoelastic half-space considering the effect of hallcurrent, input heat source and rotation subjected to inputultra-laser heat source.The micropolar theory of Thermoelasticity by Eringen (1966) has been used to investigate the problem. Normal mode analysis technique has been used to solve the resulting non–dimensional coupled field equations toobtain displacement, stress components and temperature distribution.

Keywords: Micropolar thermoelastic, Hall Current, Thermal laser heat source, rotation.

Introduction:

The problems involving the investigation of effect of magnetic field (that may be earth's magnetic field or other human generated high intensity magnetic field) and thermal loading by lasers on various type of materials are of great importance in seismological research and in engineering applications. The linear theory of micropolar elasticity was developed by Eringen [1]. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. Rigid chopped fibers, elastic solids with rigid granular inclusions and other industrial materials such as liquid crystals are examples of such materials. Zakaria [2]investigated the effects of Hall current and rotation on magneto micropolargeneralized Thermoelasticityincluding the boundary condition with a source of ramp type heating. Scruby et al. [3]investigateda mathematical model of point source to study the ultrasonic evolution by lasers. He studied the physics of heated plate by laserheat loading in the thermoelastic system as a surface center of expansion (SCOE). Also forone-pointlaser heat inputRose [4]provided more accurate mathematical basis. Later McDonald [5] and Spicer [6]gave a mathematical model known as laser-generated ultrasound modelby introducing thermo-diffusion concept. Dubois [7]verified by experimental results that penetration depth playsan important role in the generation of laser-ultrasound. Abo-Dahab and Abbas [8] investigated LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity.Chen et al. [9] and Kim et al. [10]investigated some other such type of research. Thermoelastic behavior of laser heatin context of different theories of thermoelasticity was presented by Youssef and Al-Bary [11]. A 2- dimensional problem in generalized thermoelastic medium with thermo-diffusion was investigated by Elhagary[12]. Kumar et al.[13] studied the elastodynamical interactions of input heat source with microstretch thermoelastic medium.

Basic equations

Following Eringen [1], Al Qahtani and Dutta [14]and Zakaria [2] are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T+\mu_0\epsilon_{rji}J_rH_j=\rho\left(\ddot{\boldsymbol{u}}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})+2\boldsymbol{\Omega}\times\frac{\partial u}{\partial t}\right) \quad ,(1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\left(\ddot{\boldsymbol{\phi}}+\Omega\times\frac{\partial\boldsymbol{\phi}}{\partial t}\right), \quad (2)$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\beta_1T_0(\nabla.\dot{\boldsymbol{u}}-Q)+\nu_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*, \quad (3)$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\epsilon_{ijk}\phi_k\right)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T, \quad (4)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i} \quad (5)$$

The current density vector $\boldsymbol{J}$ can be expressed as:

$$\boldsymbol{J}=\frac{\sigma_0}{1+m^2}\left[\boldsymbol{E}+\mu_0(\dot{\boldsymbol{u}}\times\boldsymbol{H})-\frac{\mu_0}{en_e}(\boldsymbol{J}\times\boldsymbol{H})\right]. \quad (6)$$

Here$\boldsymbol{F}=\mu_0(\boldsymbol{J}\times\boldsymbol{H})$ is the Lorentz force, $\boldsymbol{H}$ is the magnetic field vector, $\boldsymbol{E}$ is the intensity of electric field, m is the Hall parameter,σ_0 is the electrical conductivity, e is the charge of an electron, $n_e$is the number density of electrons. Further the plate surface is illuminated by input heat given by the heat input

$$Q=I_0f(t)g(x_1)h(x_3), \quad (7)$$

$$f(t)=\frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)},(8)$$

$$g(x_1)=\frac{1}{2\pi r^2}e^{-\left(\frac{x_1^2}{r^2}\right)}, \quad (9)$$

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$$h(x_3) = \gamma^* e^{-\gamma^* x_3} \tag{10}$$

$where,$ I_0energy absorbed, t_0 is the pulse rising time,r is the beam radius.

Equation (7) with substitution of (8- 10) takes the form

$$Q = \frac{I_0 \gamma^*}{2\pi r^2 t_0^2} t e^{-\left(\frac{t}{t_0}\right)} e^{-\left(\frac{x_1^2}{r^2}\right)} e^{-\gamma^* x_3} \tag{11}$$

Formulation of the problem:

We consider plane strain problem with all the field variables depending on (x_1, x_3, t). For two dimensional problems, we take

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \tag{12}$$

For further consideration, it is convenient to introduce in equations (1)-(3) the dimensionless quantities defined as:

$$x_i' = \frac{\omega^*}{c_1} x_i \,,\; u_i' = \frac{\rho \omega^* c_1}{\beta_1 T_0} u_i , \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0} \phi_i \,, T' = \frac{T}{T_0} \,, t' = \omega^* t \,, \tau_1' = \omega^* \tau_1 \,, \tau_0' = \omega^* \tau_0 \,, t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij} \,,$$

$$\omega^* = \frac{\rho c^* c_1^2}{K^*} \,,\; c_1^2 = \frac{\lambda + 2\mu + k}{\rho} \tag{13}$$

Making use of equation (12)-(13) the system of equations (1)-(3) reduces to:

$$\zeta_1 \frac{\partial e}{\partial x_1} + \zeta_2 \nabla^2 u_1 - \zeta_3 \frac{\partial \phi_2}{\partial x_3} + {\Omega_0}^2 u_1 - 2\Omega_0 \frac{\partial u_3}{\partial t} + \frac{M}{1+m^2}\left(\frac{\partial u_1}{\partial t} + m \frac{\partial u_3}{\partial t}\right) - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \ddot{u}_1, \tag{14}$$

$$\zeta_1 \frac{\partial e}{\partial x_3} + \zeta_2 \nabla^2 u_3 + \zeta_3 \frac{\partial \phi_2}{\partial x_1} + 2\Omega_0 \frac{\partial u_1}{\partial t} + {\Omega_0}^2 u_3 - \frac{M}{1+m^2}\left(m \frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t}\right) - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = \ddot{u}_3, \tag{15}$$

$$\nabla^2 \phi_2 - 2\zeta_4 \phi_2 + \zeta_4 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = \zeta_5 \ddot{\phi}_2, \tag{16}$$

$$-\nabla^2 T + \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \zeta_6 \left(1 + \varepsilon \tau_0 \frac{\partial}{\partial t}\right)(\dot{e} - Q) = Q_0 f^*(x_1, t) e^{-\gamma^* x_3}, \tag{17}$$

Using Helmholtz's theorem, the displacement components u_1 and u_3 are related to the non-dimensional potential functions ϕ and ψ by the relation mentioned below:

$$u_1 = \frac{\partial \phi}{\partial x_1} + \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} - \frac{\partial \psi}{\partial x_1} \tag{18}$$

Substituting the values of u_1 and u_3 from (18) in (14)-(17), we obtain:

$$\left(\nabla^2 + \Omega^2 + \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\phi + \left(2\Omega - \frac{mM}{1+m^2}\right)\frac{\partial \psi}{\partial t} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)T = 0, \tag{19}$$

$$\left(a_3\nabla^2 + \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\psi - \left(2\Omega - \frac{mM}{1+m^2}\right)\frac{\partial \phi}{\partial t} - a_4\phi_2 = 0, \tag{20}$$

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\dot{T} + a_5\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi - \nabla^2 T = Q_0 f^*(x_1, t)e^{-\gamma^* x_3}, \tag{21}$$

$$\left(\nabla^2 - 2a_1 + a_2\frac{\partial^2}{\partial t^2}\right)\phi_2 + a_1\nabla^2\psi = 0 \tag{22}$$

Solution of the problem

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi, \psi, T, \phi^*\}(x_1, x_3, t) = \{\bar{\phi}, \bar{\psi}, \bar{T}, \overline{\phi^*}\}(x_3)e^{i(kx_1 - \omega t)}. \tag{23}$$

Making use of (23) in equations (19)-(22) and after some simplifications, yield:

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{\phi} = f_1(\gamma^*, x_1, t)e^{-\gamma^* x_3}, \tag{24}$$

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{T} = f_2(\gamma^*, x_1, t)e^{-\gamma^* x_3}, \tag{25}$$

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{\phi}_2 = f_3(\gamma^*, x_1, t)e^{-\gamma^* x_3}, \tag{26}$$

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{\psi} = f_4(\gamma^*, x_1, t)e^{-\gamma^* x_3}. \tag{27}$$

The solution are given as following:

$$\bar{\phi} = \sum_{i=1}^{4} c_i e^{-m_i x_3} + \frac{f_1}{f_5} e^{-\gamma^* x_3}, \tag{28}$$

$$\bar{T} = \sum_{i=1}^{4} \alpha_i c_i e^{-m_i x_3} + \frac{f_2}{f_5} e^{-\gamma^* x_3}, \tag{29}$$

$$\overline{\phi_2} = \sum_{i=1}^{4} \beta_i c_i e^{-m_i x_3} + \frac{f_3}{f_5} e^{-\gamma^* x_3}, \tag{30}$$

$$\bar{\psi} = \sum_{i=1}^{4} \delta_i c_i e^{-m_i x_3} + \frac{f_4}{f_5} e^{-\gamma^* x_3}. \tag{31}$$

Here $m_i^2 (i = 1,2,3,4)$ are the roots of characteristic equation of equation (24).

Substituting the values of $\bar{\phi}, \bar{T}, \bar{\phi}_2, \bar{\psi}$ from the equations (28)-(31) in the (4)-(5), and using (12)-(13), (18) and solving the resulting equations, we obtain:

$$\bar{t}_{33} = \sum_{i=1}^{4} G_{1i} e^{-m_i x_3} - M_1 e^{-\gamma^* x_3}, \tag{32}$$

$$\bar{t}_{31} = \sum_{i=1}^{4} G_{2i} e^{-m_i x_3} - M_2 e^{-\gamma^* x_3}, \tag{33}$$

$$\bar{m}_{32} = \sum_{i=1}^{4} G_{3i} e^{-m_i x_3} - M_3 e^{-\gamma^* x_3}, \tag{34}$$

$$\bar{T} = \sum_{i=1}^{4} G_{4i} e^{-m_i x_3} - M_4 e^{-\gamma^* x_3} \tag{35}$$

$G_{mi} = g_{mi} C_i, i = 1,2,...,4$. g_{mi} and M_i are mentioned in appendix A.

Boundary Conditions:

We consider normal and tangential forces actingat the surface $x_3 = 0$ along with vanishing of couple stressat $x_3 = 0$ and $I_0 = 0$. Mathematically this can be written as:

$$t_{33} = -F_1 e^{-(kx_1 - \omega t)}, t_{31} = -F_2 e^{-(kx_1 - \omega t)}, m_{32} = 0, \frac{\partial T}{\partial x_3} = 0 \tag{36}$$

where F_1 and F_2 are the magnitude of the applied forces.

Substituting the expression of the variables considered into these boundary conditions, we can obtain the following equations:

$$\sum_{i=1}^{4}(g_{1i}, g_{2i}, g_{3i}, g_{4i})c_i = (-F_1, -F_2, 0, 0). \quad (37)$$

Conclusions:

The new model is employed in magneto-micropolar thermoelastic medium as a new improvementin the field of Thermoelasticity. The subject becomes more interesting due to Hall current involving rotation and irradiation of an ultra-laser pulsewith an extensive short duration or a very high heat flux. This type of problems has found numerous applications. The methodused in this article is applicable to a wide range of problems in thermodynamics. By the obtained results,it is expected that the present model of equations will serve as more realistic and will provide motivationto investigate micropolar Thermoelasticity problems.

Conflict of Interest:

The authors declare that there is no conflict of interest regarding the publication of this paper.

References:

[1] A.C. Eringen,“Linear theory of micropolar elasticity,” *Indiana University Mathematics Journal*,**15**, 909-923, 1966.

[2] M. Zakaria, “Effects of Hall Current and Rotation on Magneto-Micropolar Generalized Thermoelasticity Due to Ramp-Type Heating,” *International Journal of Electromagnetics and Applications*,**2,** 24-32, 2012.

[3] C. B. Scruby and L. E. Drain, *Laser Ultrasonics Techniques and Applications*, Adam Hilger, Bristol, UK, 1990.

[4] L.R.F. Rose, “Point-source representation for laser-generated ultrasound,”*Journal of the Acoustical Society of America*,**75**, 3, 723, 1984.

[5] F. A. McDonald, “On the Precursor in Laser-Generated Ultrasound Waveforms in Metals,”*Applied Physics Letters*, **56**, 3, 230-232, 1990.

[6] J. B. Spicer et al., “Quantitative Theory for Laser Ultrasonic Waves in a Thin Plate,”*Applied Physics Letters*, **57**, 1882–1884, 1990.

[7] M. Dubois et al., “Modelling of laser thermoelastic generation of ultrasound in an orthotropic medium,”*Applied Physics Letters*, **64**, 554, 1994.

[8] S.M. Abo-Dahab et al.,“LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity,”*Applied Mathematical Modelling*.**35**, 3759-3768, 2011.

[9] J.K. Chen et al., “Comparison of one-dimensional and two-dimensional axisymmetric approaches to the thermomechanical response caused by ultrashort laser heating,”*Journal of Optics*, **4**, 650-661, 2002.

[10] W.S. Kim et al., “Thermoelastic stresses in a bonded layer due to repetitively pulsed laser heating,”*Acta Mechanica*, **125**, 107-128, 1997.

[11] H.M. Youssef et al., "Thermoelastic material response due to laser pulse heating in context of four theorems of thermoelasticity,"*Journal of thermal stresses*, **37**, 1379-89, 2014.

[12] M.A. Elhagary, "A two-dimensional generalized thermoelastic diffusion problem for a thick plate subjected to thermal loading due to laser pulse,"*Journal of thermal stresses*, **37**, 1416-1432, 2014.

[13] R. Kumar et al., "Thermomechanical interactions of Laser Pulse with Microstretch Thermoelastic Medium,"*Archives of Mechanics*, 67(6), 439-456, 2015.

[14] M.H. Al-Qahtani et al., "Laser-generated thermoelastic waves in an anisotropic infinite plate: Exact analysis,"*Journal of Thermal Stresses*, 31, 2008, 569-583.

[15] A.C. Eringen,*Microcontinuum Field Theories I: Foundations and Solids*. New York: Springer-Verleg, 1999.

[16] R.S. Dhaliwal, A. Singh,*Dynamic Coupled Thermoelasticity.* New Delhi: Hindustan Publication Corporation, 1980.