International Conference on Latest Innovation in Engineering Science and Management
Buddha Institute of Technology, GIDA, Gorakhpur (UP)
30th - 31st March 2019 www.conferenceworld.in ISBN : 978-93-87793-80-4

Proof of Closure Properties of Regular Languages Using Myhill-Nerode Theorem

Shrawan Kumar Pandey[#1], Rajeev Ranjan Kumar Tripathi [*2], Manish Kumar Gupta[3]

[#]Computer Science & Engineering Department, Buddha Institute of Technology
GIDA, Gorakhpur
Affiliated to AKTU Lucknow, India

Abstract—Closure properties: This is very important properties for formal languages used in automata theory. A certain type of language under some operation yield same type of language, then it is said to be language is closed under that operation. Example is that regular language under union, intersection, concatenation and some others is closed.
Myhill-Nerode Theorem: It is very important theorem for determining a given language is regular or not, because it gives necessary and sufficient condition for a given language to be regular language, on the basis of finite number of equivalence classes.

***Keywords*— Closure properties, Formal languages, Union, Intersection, Concatenation, Myhill-Nerode theorem, Regular language, Equivalence classes.**

I. INTRODUCTION

Language L is a regular language if and only if the set of equivalence classes of L is finite.

OR

1. Language L can be divided into set of all possible strings into separate (mutually exclusive) classes.
2. If L is a regular language then number of classes created is finite.
3. If number of classes that L has is finite, then L is a regular language.

Equivalence Classes:

Classes: Let L = (a+b)*a + (a+b)*b

There is two classes-

$C_a = (a+b)^*a$ ending in a.

$C_a = (a+b)^*b$ ending in b.

A binary relation is said to be equivalence relation if and only if it is

1. Reflexive
2. Transitive
3. Symmetric

There is a natural equivalence relation between the strings of finite automata.

Suppose DFAM= (Q, E, δ, q_0, F) for x, y, z ∈ ∑*, a relation R_m is defined as follows

xR_my iff $\delta^*(q_0, x) = (q_0, y)$

Reflexive:

$\because\ \delta^*(q_0, x) = \delta^*(q_0, y)\ \forall x \in \sum^*$

$\therefore\ xR_mx$

$\therefore\ R_m$ is reflexive.

Transitive:

$\because\ \delta^*(q_0, x) = \delta^*(q_0, y)$ and $\delta^*(q_0, y) = \delta^*(q_0, z)$

$\therefore\ \delta^*(q_0, x) = \delta^*(q_0, z)$

It means that if xR_my and yR_mz then xR_mz.

$\therefore\ R_m$ is transitive.

Symmetric:

$\because\ \delta^*(q_0, x) = \delta^*(q_0, y)$

$\therefore\ \delta^*(q_0, y) = \delta^*(q_0, x)$

$\therefore\ R_m$ is symmetric.

$\therefore\ R_m$ is equivalence classes.

The equivalence relation R_m divides the set ∑* into equivalence classes. The number of equivalence classes is known as index and it is finite always for regular language.

Now we will use these concepts in proving closure properties of regular language.

II. CLOSUER UNDER UNION

If L and M are regular language, so is L∪M.

Proof: If L and M are regular languages, they have finite

number of classes and are C_L and C_M respectively.

Therefore C_L and C_M are finite.

Let L∪M have number of classes is C_U, then

$\max(C_L, C_M) \le C_U \le C_L + C_M$

(Using set theory union concept)

Therefore

C_U have four possible values :

a. C_L
b. C_M
c. $C_L + C_M$
d. any value between C_L or C_M and $C_L + C_M$

All four values are finite.

So number of classes in L∪M is finite.

∴ L∪M is regular language.

III. CLOSURE UNDER INTERSECTION

If L and M are regular language, then so is L⋂M.

Proof: If L and M are regular languages, they have finite number of classes and are C_L and C_M respectively.

Therefore C_L and C_M are finite.

Let L⋂M have number of classes is $C_\cap$, then

$$0 \leq C_\cap \leq \min(C_L, C_M)$$

(Using set theory intersection concept)

So $C_\cap$ is finite. Therefore L⋂M is regular.

IV. CLOSURE UNDER CONCATENATION

If L and M are regular language, then so is LM.

Proof: If L and M are regular languages, they have finite number of classes and are C_L and C_M respectively.

Let LM have number of classes C_{LM}

Therefore

$$C_L \text{ or } C_M \leq C_{LM} \leq C_L.C_M$$

(Using Cartesian product concept)

i.e. finite so LM is regular language.

V. CLOSURE UNDER KLEENE CLOSURE

If L is regular language so is L*.

Let L is regular language having finite number of classes i.e.C_L.

Proof: $C_* = C_L + 1$

$\because$ C_L is finite.

$\therefore$ C_* is finite.

And so L* is regular language.

VI. CLOSURE UNDER DIFFERENCE

If L and M are regular language, then so is L-M.

Proof: Let L, M and L-M have C_L, C_M, and C_{L-M} number of classes.

Therefore

$$C_L \leq C_{L-M} \leq C_L - C_M$$

(Using set theory concept)

i.e. finite so L-M is regular language.

VII. CLOSURE UNDER COMPLEMENTATION

The complement of a language L (w.r.t. an alphabet ∑ such that ∑* contains L*) is ∑*- L.

Since ∑* is regular language, the complement of a regular language is always regular.

Proof: Let ∑ have n finite element.

Let ∑* have number of classes $C_{\sum*}$.

$\therefore$ $C_{\sum*} = n+1$

Let ∑*- L have number of classes $C_{\sum*-L}$.

Then $C_{\sum*-L} = (n+1) - C_L$

(Using set theory concept)

i.e. finite.

(where C_L is number of classes in L)

Therefore complement of L is regular.

VIII. CLOSURE UNDER REVERSAL

If L is regular then L^R is also regular.

If number of classes in L is C_L, then number of classes in L^R is also C_L.

Therefore L^R is regular.

We can also prove other properties in same way.

IX. CONCLUSION

It is very easy to prove closure properties using Myhill-Nerode theorem with the help of set theory and Cartesian product concept. We have also tried to explore importance of Myhill-Nerode theorem beyond the proving of given language is regular language.

REFERENCES

[1] Henzinger, Tom (2003), Lecture 7: Myhill–Nerode Theorem.

[2]Hopcroft, John E.; Ullman, Jeffrey D. , “ Introduction to Automata Theory, Languages, and Computation,” Addison-Wesley Publishing, ISBN0-201-02988-X.

[3]Nerode, Anil , "Linear Automaton Transformations", Proceedings of the AMS 9, JSTOR 2033204.

[4] Regan, Kenneth (2007), Notes on the Myhill-Nerode Theorem.