

Sparse technique for estimating Direction of Arrival by employing orthogonal matching pursuit

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ABSTRACT

Array signal processing has wide applications, such as radar, wireless communication, sonar, radio astronomy and navigation. In array signal processing the problem we face is determining the direction of signal source at the receiving antenna.

This paper projects the simulation of orthogonal matching pursuit using sparse technique concepts adding white noise and interference noise. The simulation in this paper shows the performance of DOA estimation algorithm depends on factors such as number of signals, number correlation between the signal source, number of array elements, angle between adjacent signal source and array element spacing.

INTRODUCTION

Direction of Arrival (DOA) denotes the direction from which usually a propagation wave arrives at a point. DOA estimation of far field narrow band signal has been of interest in the past few decades[1]. There are many DOA estimation techniques such as Capon/Minimum Variance Distortion-less Response(MVDR), Beam forming, Root-Multiple Signal Classification(MUSIC), Improved-MUSIC and ESPRIT[2].

Sparse Approximation is also known as Sparse Representation. Sparse Representation theory deals with system of linear equation of sparse solution[3]. Techniques for finding the solutions

and exploiting them in applications they can be used in image processing, signal processing, machine learning, medical imaging and many more. Sparse Approximation has gained a good reputation in theoretical research and practical application. For sparse representation different algorithms have been proposed[3].

Signal Model for DOA

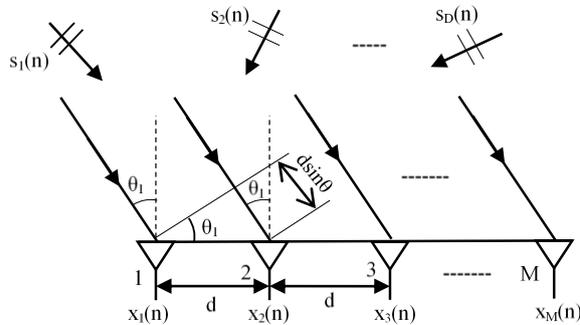
The data model used for DOA estimation problem assumes that the received signal which is impinging on the array of sensors is emitted by a far-field point source and is narrowband. Considering a uniform linear array (ULA)[2] which consists of M sensors which are equi-spaced, on these sensors narrowband plane waves from multiple signal sources D are impinging as shown in the Fig 1.

At any particular instance of time i.e $n = 1, 2, \dots, K$, where the total number of snapshots taken is represented by K and the output of m^{th} sensor is given by (1).

$$x_m(n) = s_1(n)e^{-j\beta d(m-1)\sin\theta_1} + s_2(n)e^{-j\beta d(m-1)\sin\theta_2} + \dots + s_D(n)e^{-j\beta d(m-1)\sin\theta_D} + w_m \quad (1)$$

Where the D source signals $s_1(n), s_2(n), \dots, s_D(n)$ which are impinging on the uniform linear array with the angle of arrival of $\theta_1, \theta_2, \dots, \theta_D$ respectively, $\beta = 2\pi/\lambda$, here the wavelength of the received signal is λ , the spacing between the array elements is given by d , the noise of the m^{th} sensor is given by $w_m(n)$.

Representing (1) for all the M sensor outputs



Where $[.]^H$ represents the Hermitian transpose of a

inmatrix form gives equation (2).

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{W} \quad (2)$$

Where the $M \times 1$ received signal vector is, $\mathbf{X} = [x_1(n) \ x_2(n) \ \dots \ x_M(n)]^T$, the array steering matrix \mathbf{A} which has Vandermonde structure which is given in (3), the $D \times 1$ signal source vector is $\mathbf{S} = [s_1(n) \ s_2(n) \ \dots \ s_D(n)]^T$, the $M \times 1$ sensor generated noise vector is $\mathbf{W} = [w_1(n) \ w_2(n) \ \dots \ w_M(n)]^T$.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j\beta d \sin \theta_1} & e^{-j\beta d \sin \theta_2} & \dots & e^{-j\beta d \sin \theta_D} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\beta d (M-1) \sin \theta_1} & e^{-j\beta d (M-1) \sin \theta_2} & \dots & e^{-j\beta d (M-1) \sin \theta_D} \end{bmatrix} \quad (3)$$

In the above matrix \mathbf{A} each column of this matrix represents the array response vector for a particular incident signal. Let us take an example to explain this further, in the matrix \mathbf{A} the first column is given as in (4) represents the array response vector for the first incident signal which has an angle of arrival θ_1 .

$$\mathbf{a}(\theta_1) = [1 \ e^{-j\beta d \sin \theta_1} \ \dots \ e^{-j\beta d (M-1) \sin \theta_1}]^T \quad (4)$$

Assuming that both the sensor noise $w(n)$ and the received signal $x(n)$ are highly uncorrelated, the sensor noise $w(n)$ can be temporarily modeled as white and zero mean complex Gaussian process. \mathbf{R}_{xx} is the spatial auto-correlation matrix of the received signal vector \mathbf{X} and this can be defined as in equation (5)

$$\mathbf{R}_{xx} = E\{\mathbf{X}\mathbf{X}^H\} \quad (5)$$

The auto-correlation matrices are unknown in real-time measurements; therefore, finite numbers of data samples called snapshots are used to estimate the auto-correlation matrices. The natural estimate of auto-correlation matrix is given by (6).

$$\hat{\mathbf{R}}_{xx} = \sum_{n=1}^K \mathbf{X}(n)\mathbf{X}^H(n)$$

matrix; K represents the number of snapshots taken

Orthogonal Matching Pursuit Algorithm

The equation for generating sparse signal is

$$\mathbf{A}\mathbf{X} = \mathbf{Y}$$

Where, \mathbf{A} is the measurement matrix generated using IDFT.

Here, \mathbf{Y} is the measurement vector. When random samples are taken from a generated matrix from a certain equation as per the requirements it is a measurement vector.

The sparse signal to be generated is \mathbf{X} .

We are using OMP algorithm to generate a sparse signal \mathbf{X} .

Step 1: Let us take an empty matrix into consideration and call it matrix \mathbf{K} . The positions of the nonzero values in sparse signal are stored in \mathbf{K} .

Step 2: In e take a replica of the measurement matrix.

$$e = \mathbf{Y}$$

Step 3: Take $\epsilon = 0.01$

Step 4: Until the residue value, r becomes less than ϵ keep doing multiple iterations.

Step 5: To find residue value

Let the columns of the measurement matrix \mathbf{A} be b_1, b_2, \dots, b_n .

Multiplying b_1, b_2, \dots, b_n with Y separately

We get $X_k = X_1 X_2 \dots X_n$

Where,

$$\begin{aligned} X_1 &= b_1 * Y; \\ X_2 &= b_2 * y; \\ X_n &= b_n * y; \end{aligned}$$

In matlab we can calculate X_k using $X_k = \text{pinv}(A) * Y$.

Assuming X_1 has the highest value, while neglecting the negative values. It contributes the maximum value towards Y .

Now calculate the residue value

$$r = Y - X_1 * b_1$$

Now if the residue value is greater than ϵ then continue onward for second interaction taking b_2, b_3 and so on forth.

Step 6: During the iteration, the position which is contributing the maximum value is stored in matrix K .

Step 7: Generate a $N \times 1$ matrix X with all zeros in it.

Step 8: Now for all the positions not in K , put $X=0$.

For all positions in K , put $X=X_k$.

OMP Algorithm Table

Input: • Measurement vector y
 • Measurement matrix A
 • Number of selected coefficients in each Iteration r ,
 By default $r = 1$
 • Required precision ϵ

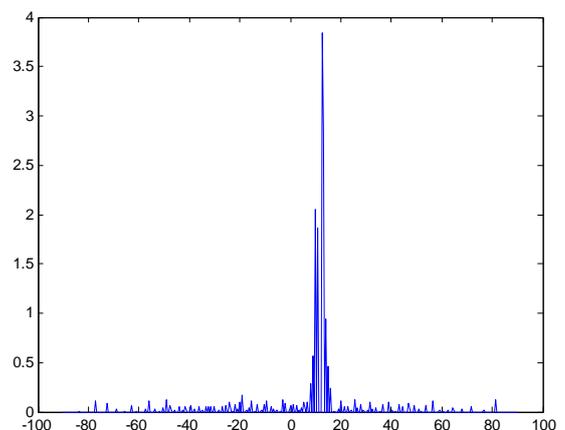
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1:  $K \leftarrow \emptyset$ 
2:  $e \leftarrow y$ 
3: while  $\|e\|_2 > \epsilon$  do
4:  $(k_1, k_2, \dots, k_r) \leftarrow$  positions of  $r$  highest values in  $AH^T e$ 
5:  $K \leftarrow K \cup \{k_1, k_2, \dots, k_r\}$ 
6:  $AK \leftarrow$  columns of matrix  $A$  selected by set  $K$ 
7:  $XK \leftarrow \text{pinv}(AK) y$ 
8:  $y_K \leftarrow AK XK$ 
9:  $e \leftarrow y - y_K$ 
10: end while
11:  $X \leftarrow \begin{cases} 0 & \text{for positions not in } K \\ XK & \text{for positions in } K \end{cases}$ 
    
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Output: • Reconstructed signal coefficients X .

Experimental Results

Final Output:



As you can see in the above figure, The peak show the reconstructed original signals.

Reference

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