International Conference on Contemporary Multidisciplinary Research and Innovation
Date: 28th March 2026
ISBN: 978-93-48954-63-3

CERTAIN CLASSES OF INTUITIONISTIC FUZZY OPEN SETS AND THEIR RELATIONSHIPS

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Abstract:

Fuzzy set theory, introduced by Lotfi A. Zadeh in 1965, has played an important role in modeling uncertainty in mathematical systems. The extension of this theory to fuzzy topology was proposed by C. L. Chang in 1968. Later, intuitionistic fuzzy sets were introduced by K. T. Atanassov in 1983, incorporating both membership and non-membership degrees. The concept of intuitionistic fuzzy topological spaces was subsequently developed by D. Çoker. In this paper, we study intuitionistic fuzzy open sets and analyze their relationships within intuitionistic fuzzy topology. Several properties of these sets are discussed, and their connections with other related intuitionistic fuzzy sets are examined.

Keywords: Intuitionistic Fuzzy Sets, Intuitionistic Fuzzy Topological Spaces, Intuitionistic Fuzzy Open Sets, Fuzzy Topology, Relationships of Intuitionistic Fuzzy Sets.

1. INTRODUCTION

The study of uncertainty and vagueness in mathematical systems gained significant attention after the introduction of fuzzy set theory by L. A. Zadeh in 1965. In this theory, elements can belong to a set with varying degrees of membership rather than in a strict binary form as in classical set theory. This concept provided a powerful mathematical framework for representing imprecise information and has been widely applied in various scientific and engineering fields [1]. The development of fuzzy set theory also encouraged further research in topology. In particular, C. L. Chang introduced the concept of fuzzy topological spaces in 1968, extending classical topological notions such as open sets and continuity to the fuzzy environment [2].

ISBN: 978-93-48954-63-3

Although fuzzy sets effectively describe partial membership, they do not explicitly represent the degree of non-membership or the uncertainty arising from incomplete information. To address this limitation, K. T. Atanassov introduced the concept of intuitionistic fuzzy sets in 1983. In this framework, each element is associated with both a membership degree and a non-membership degree satisfying the condition $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$. The remaining value represents the hesitation degree, which reflects the uncertainty regarding the membership of an element in the set [3], [4]. This additional parameter makes intuitionistic fuzzy sets a more flexible and comprehensive model for dealing with uncertainty.

The development of intuitionistic fuzzy sets naturally led to the extension of topological structures in the intuitionistic fuzzy environment. In this direction, D. Çoker introduced the concept of intuitionistic fuzzy topological spaces during 1995–1998, where classical topological notions such as open sets, closed sets, interior, closure, and continuity were generalized within the intuitionistic fuzzy framework [5], [6]. Since then, intuitionistic fuzzy topology has attracted considerable attention from researchers and has been studied extensively in the literature [7], [8].

Among the various concepts in topology, open sets play a fundamental role, as they serve as the foundation for defining several important topological properties and mappings. In intuitionistic fuzzy topological spaces, intuitionistic fuzzy open sets form the basic structure through which many other concepts are defined. Consequently, several generalized forms of intuitionistic fuzzy open sets have been introduced by different researchers in order to investigate the structural behavior of these spaces in greater detail [9]–[11].

Motivated by these developments, the present paper focuses on the study of certain classes of intuitionistic fuzzy open sets and investigates their fundamental properties in intuitionistic fuzzy topological spaces. The relationships between these sets and other related classes of intuitionistic fuzzy sets are also examined. The results obtained contribute to the systematic development of generalized openness and provide further insight into the structure of intuitionistic fuzzy topological spaces.

2. PRELIMINARIES

Definition 2.1: Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is defined as $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$. Where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote,

ISBN: 978-93-48954-63-3

respectively, the degree of membership $\mu_A(x)$ and the degree of non-membership $\gamma_A(x)$ of an element $x \in X$ in the set A. These functions satisfy the condition $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$, $\forall\, x \in X$. The quantity $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ is referred to as the degree of indeterminacy (or hesitation margin) of the element x with respect to A.

Definition 2.2: Let X be a non–empty set, and let A and B be two intuitionistic fuzzy sets in X of the forms $A = \{\langle x,\ \mu_A(x),\ \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x,\ \mu_B(x),\ \gamma_B(x)\rangle : x \in X\}$ and let $\{A_i : i \in J\}$ denote an arbitrary family of intuitionistic fuzzy sets in X. Then the following operations and relations are defined as follows:

(a) **Inclusion:** $A \subseteq B$ if and only if $\forall x \in X$, $[\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)]$.

(b) **Equality:** $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) **Complement:** $A^c = \left\{\langle x,\ \gamma_A(x),\ \mu_A(x)\rangle : x \in X\right\}$

(a) **Intersection:** $\bigcap_{i \in J} A_i = \left\{\langle x,\ \bigwedge_{i \in J} \mu_{A_i}(x),\ \bigvee_{i \in J} \gamma_{A_i}(x)\rangle : x \in X\right\}$

(b) **Union:** $\bigcup_{i \in J} A_i = \left\{\langle x,\ \bigvee_{i \in J} \mu_{A_i}(x),\ \bigwedge_{i \in J} \gamma_{A_i}(x)\rangle : x \in X\right\}$

(d) **Null and Universal intuitionistic Fuzzy Sets :** $\underline{0} = \{\langle x,\ 0,\ 1\rangle : x \in X\}$ and $\underline{1} = \{\langle x,\ 1,\ 0\rangle : x \in X\}$.

Definition 2.3: Let X be a nonempty set. A family $\Im$ of intuitionistic fuzzy sets in X is said to form an intuitionistic fuzzy topology on X if the following axioms are satisfied:

(a) **Boundary Conditions:** $\underline{0}, \underline{1} \in \Im$, Where $\underline{0} = \{\langle x,\ 0,\ 1\rangle : x \in X\}$ and $\underline{1} = \{\langle x,\ 1,\ 0\rangle : x \in X\}$.

(b) **Finite Intersection:** For any $G_1, G_2 \in I$, $G_1 \cap G_2 \in I$

(c) **Arbitrary Union:** For any arbitrary family $\{G_i : i \in J\} \subseteq I$. $\cup G_i \in I$.

In this case, the pair (X, I) is called an intuitionistic fuzzy topological space, and every intuitionistic fuzzy set belonging to I is called an intuitionistic fuzzy open set in X.

Definition 2.4: An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, I) is said to be intuitionistic fuzzy closed if its complement A^c is an intuitionistic fuzzy open set; that is, $A^c \in I$

Definition 2.5: Let (X, I) be an intuitionistic fuzzy topological space and let $A = \{\langle x,\ \mu_A(x),\ \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set in X. Then

(a) **Intuitionistic fuzzy interior**: The intuitionistic fuzzy interior of A, denoted $\text{int}(A)$, is

ISBN: 978-93-48954-63-3

defined as the largest intuitionistic fuzzy open set contained in A. Formally, int (A) = ∪{G : G is an intuitionistic fuzzy open set in X and G ⊆ A}.

(b) **Intuitionistic fuzzy closure:** The intuitionistic fuzzy closure of A, denoted Cl(A), is defined as the smallest intuitionistic fuzzy closed set containing A. That is, Cl (A) = ∩{K : K is an intuitionistic fuzzy closed set in X and A ⊆ K},

3. INTUITIONISTIC FUZZY OPEN SETS

Definition 3.1: Let (X, I) be an intuitionistic fuzzy topological space and let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set in X. Then A is said to be an intuitionistic fuzzy semi-open set if $A \subseteq cl(int(A))$. Where $int(A)$ and $cl(A)$ denote, respectively, the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A.

Example 3.1: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.3}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.5}, \frac{b}{0.3}\right)\right\}$. Clearly $\lambda \subseteq cl(int(\lambda))$, Therefore, λ is an intuitionistic fuzzy semi-open set in the intuitionistic fuzzy topological space (X, I).

Proposition 3.1: Every intuitionistic fuzzy open set is an intuitionistic fuzzy semi-open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy open set in (X, I). By definition of interior $int\ (\lambda) = \lambda$. We know, $int\ (\lambda) \subseteq \lambda$ and $\lambda \subseteq cl\ (\lambda)$. Now we have $\lambda = int\ (\lambda) \subseteq cl\left(int\ (\lambda)\right)$. Therefore $\lambda \subseteq cl\left(int\ (\lambda)\right)$. Hence. λ is an intuitionistic fuzzy semi-open set.

Remark 3.1: Converse of Proposition 3.1 is not true in general as shown in Example 3.2.

Example 3.2: In Example 3.1, λ is an intuitionistic fuzzy semi-open set but not an intuitionistic fuzzy open set in X.

Definition 3.2: Let (X, I) be an intuitionistic fuzzy topological space and let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set in X. Then A is said to be an intuitionistic fuzzy pre-open set if $A \subseteq int(cl(A)$. Where $int(A)$ and $cl(A)$ denote, respectively, the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A.

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Example 3.3: In Example 3.1, λ is an intuitionistic fuzzy pre-open set in the intuitionistic fuzzy topological space (X, I).

Proposition 3.2: Every intuitionistic fuzzy open set is an intuitionistic fuzzy pre-open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological Space and λ be any intuitionistic fuzzy open Set in (X, I). By definition of interior $\text{int}(\lambda) = \lambda$. We know $\text{int}(\lambda) \subseteq \lambda \subseteq \text{cl}(\lambda)$. Now we have $\text{int}(\lambda) \subseteq \text{int}(\text{cl}(\lambda))$. Therefore $\lambda = \text{int}(\lambda) \subseteq \text{int}(\text{cl}(\lambda))$. Hence. λ is an intuitionistic fuzzy pre-open set.

Remark 3.2: Converse of Proposition 3.2 is not true in general as shown in Example 3.4.

Example 3.4: In Example 3.1, λ is an intuitionistic fuzzy pre-open set but not an intuitionistic fuzzy open set in X.

Definition 3.3: Let (X, I) be an intuitionistic fuzzy topological space and let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set in X. Then A is said to be an intuitionistic fuzzy α-open set if $A \subseteq \text{int}\left(\text{cl}\left(\text{int}(A)\right)\right)$. Where $\text{int}(A)$ and $\text{cl}(A)$ denote, respectively, the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A.

Example 3.5: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological Space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.2}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.75}, \frac{b}{0.2}\right)\right\}$. Clearly $\lambda \subseteq \text{int}\left(\text{cl}\left(\text{int}(\lambda)\right)\right)$. Therefore, λ is an intuitionistic fuzzy α-open set in the intuitionistic fuzzy topological space(X, I).

Proposition 3.3: Every intuitionistic fuzzy open set is an intuitionistic fuzzy α--open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological Space and λ be any intuitionistic fuzzy open set in (X, I) then $\text{int}(\lambda) = \lambda$. We know $\text{int}(\lambda) \subseteq \lambda \subseteq \text{cl}(\lambda)$. Now we have $\lambda = \text{int}(\lambda) \subseteq \text{cl}(\text{int}(\lambda))$. Therefore $\lambda \subseteq \text{int}\left(\text{cl}\left(\text{int}(\lambda)\right)\right)$. Hence. λ is an intuitionistic fuzzy α-open set.

Remark 3.3: Converse of Proposition 3.3 is not true in general as shown in Example 3.6.

Example 3.6: In Example 3.5, λ is an intuitionistic fuzzy α-open set but not intuitionistic fuzzy open set in X.

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Definition 3.4: Let (X, I) be an intuitionistic fuzzy topological space and let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set in X. Then A is said to be an intuitionistic fuzzy β -open set if $A \subseteq cl(int(cl(A)))$. Where $int(A)$ and $cl(A)$ denote, respectively, the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A.

Example 3.7: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.2}, \frac{b}{0.7}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.2}, \frac{b}{0.6}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.2}, \frac{b}{0.75}\right)\right\}$. Clearly $\lambda \subseteq cl(int(cl(A)))$. Therefore, λ is an intuitionistic fuzzy β-open set. in the intuitionistic fuzzy topological space(X, I).

Proposition 3.4: Every intuitionistic fuzzy open set is an intuitionistic fuzzy β--open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy open set in (X, I) By definition of interior $int\,(\lambda) = \lambda$. We know $int\,(\lambda) \subseteq \lambda \subseteq cl\,(\lambda)$. Now we have $int(\lambda) \subseteq int\,\left(cl\,(\lambda)\right)$. Therefore $\lambda \subseteq cl\,(\lambda) \subseteq cl\left(int\,\left(cl\,(\lambda)\right)\right)$. Hence. λ is an intuitionistic fuzzy β-open set.

Remark 3.4: Converse of Proposition 3.4 is not true in general as shown in Example 3.8.

Example 3.8: In Example 3.7, λ is an intuitionistic fuzzy β-open set but not intuitionistic fuzzy open set in X.

Proposition 3.5: Every intuitionistic fuzzy α-open set is an intuitionistic fuzzy semi--open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy α-open set in (X, I). By the definition of an intuitionistic fuzzy α-open set, we have $\lambda \subseteq int\left(cl\,\left(int\,(\lambda)\right)\right)$. We know $int\,(\lambda) \subseteq \lambda \subseteq cl\,(\lambda)$. Now we have $int\left(cl\,\left(int\,(\lambda)\right)\right) \subseteq cl\,\left(int\,\,(\lambda)\right)$. Therefore, $\lambda \subseteq int\left(cl\,\left(int\,(\lambda)\right)\right) \subseteq cl\,\left(int\,(\lambda)\right)$. Hence. λ is an intuitionistic fuzzy semi-open set.

Remark 3.5: Converse of Proposition 3.5 is not true in general as shown in Example 3.9.

Example 3.9: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\}$.

ISBN: 978-93-48954-63-3

Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.3}\right)\right\}$. Clearly $cl\left(int\,(\lambda)\right)$ and $\lambda \nsubseteq int\left(cl\left(int\,(\lambda)\right)\right)$. Therefore, λ is an intuitionistic fuzzy semi-open Set but not intuitionistic fuzzy α-open set in X.

Proposition 3.6: Every intuitionistic fuzzy α-open set is an intuitionistic fuzzy pre--open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy α-open set in (X, I). By the definition of an intuitionistic fuzzy α-open set, we have $\lambda \subseteq int\left(cl\left(int\,(\lambda)\right)\right)$. We know $int\,(\lambda) \subseteq \lambda \subseteq cl\,(\lambda)$. Now we have $cl\left(int\,(\lambda)\right) \subseteq cl\,(\lambda)$. Therefore $\lambda \subseteq int\left(cl\left(int\,(\lambda)\right)\right) \subseteq int\left(cl\,(\lambda)\right)$. Hence. λ is an intuitionistic fuzzy pre-open set.

Remark 3.6: Converse of Proposition 3.6 is not true in general as shown in Example 3.10.

Example 3.10: In Example 3.9, λ is an intuitionistic fuzzy pre-open set but not an intuitionistic fuzzy α-open set in X.

Proposition 3.7: Every intuitionistic fuzzy semi-open set is an intuitionistic fuzzy β--open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy semi-open set in (X, I). By the definition of an intuitionistic fuzzy semi-open set, we have $\lambda \subseteq cl\left(int\,(\lambda)\right)$ We know $int\,(\lambda) \subseteq \lambda \subseteq cl\,(\lambda)$. Now we have $int\,(\lambda) \subseteq int\left(cl\,(\lambda)\right)$. Therefore $\lambda \subseteq cl\left(int\,(\lambda)\right) \subseteq cl\left(int\left(cl\,(\lambda)\right)\right)$. Hence. λ is an intuitionistic fuzzy β-open set.

Remark 3.7: Converse of Proposition 3.7 is not true in general as shown in Example 3.11.

Example 3.11: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.3}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right\}$. Clearly $\lambda \subseteq cl(int(cl(\lambda)))$ and $\lambda \nsubseteq cl(int(\lambda))$. Therefore, λ is an intuitionistic fuzzy β-open set but not an intuitionistic fuzzy semi-open set in the intuitionistic fuzzy topological space (X, I).

Proposition 3.8: Every intuitionistic fuzzy pre-open set is an intuitionistic fuzzy β--open set, but the converse need not be true.

ISBN: 978-93-48954-63-3

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy pre-open set in (X, I). By the definition of an intuitionistic fuzzy pre-open Set, we have $\lambda \subseteq \text{int}\left(\text{cl}\,(\lambda)\right)$ We know $\text{int}\,(\lambda) \subseteq \lambda \subseteq \text{cl}\,(\lambda)$. Now we have $\text{cl}\,(\lambda) \subseteq \text{cl}\left(\text{int}\left(\text{cl}\,(\lambda)\right)\right)$. Therefore $\lambda \subseteq \text{cl}\,(\lambda) \subseteq \text{cl}\left(\text{int}\left(\text{cl}\,(\lambda)\right)\right)$. Hence. λ is an intuitionistic fuzzy β-open set.

Remark 3.8: Converse of Proposition 3.8 is not true in general as shown in Example 3.12.

Example 3.12: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.3}, \frac{b}{0.6}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.35}\right)\right\}$. Clearly $\lambda \subseteq \text{cl}(\text{int}(\text{cl}(\lambda)))$ and $\lambda \nsubseteq \text{int}(\text{cl}(\lambda))$. Therefore, λ is an intuitionistic fuzzy β-open set but not an intuitionistic fuzzy pre-open set in X.

Proposition 3.9: Every intuitionistic fuzzy α-open set is an intuitionistic fuzzy β--open set, but the converse need not be true.

Proof: Let (X, I) be an intuitionistic fuzzy topological space and λ be any intuitionistic fuzzy α-open set in (X, I). By the definition of an intuitionistic fuzzy α-open set, we have $\lambda \subseteq \text{int}\left(\text{cl}\left(\text{int}\,(\lambda)\right)\right)$. We know $\text{int}\,(\lambda) \subseteq \lambda \subseteq \text{cl}\,(\lambda)$. Now we have $\text{int}\,(\lambda) \subseteq \text{int}\left(\text{cl}\,(\lambda)\right)$. Therefore $\lambda \subseteq \text{int}\left(\text{cl}\left(\text{int}\,(\lambda)\right)\right) \subseteq \text{cl}\left(\text{int}\,(\lambda)\right) \subseteq \text{cl}\left(\text{int}\left(\text{cl}\,(\lambda)\right)\right)$. Hence. λ is an intuitionistic fuzzy β-open set.

Remark 3.9: Converse of Proposition 3.9 is not true in general as shown in Example 3.13.

Example 3.13: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.3}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right\}$. Clearly $\lambda \subseteq \text{cl}(\text{int}(\text{cl}(\lambda)))$ and $\lambda \nsubseteq \text{int}\left(\text{cl}\left(\text{int}\,(\lambda)\right)\right)$. Therefore, λ is an intuitionistic fuzzy β-open set but not an intuitionistic fuzzy α-open set in the intuitionistic fuzzy topological space (X, I).

Proposition 3.10: In an intuitionistic fuzzy topological space (X, I), the classes of intuitionistic fuzzy semi-open sets and intuitionistic fuzzy pre-open sets are independent.

ISBN: 978-93-48954-63-3

Example 3.14: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.3}, \frac{b}{0.6}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.35}, \frac{b}{0.5}\right)\right\}$. Clearly $\lambda \subseteq cl(int(\lambda))$ and $\lambda \nsubseteq int(cl(\lambda))$. Therefore, λ is an intuitionistic fuzzy semi-open set but not an intuitionistic fuzzy pre-open set in the intuitionistic fuzzy topological space (X, I).

Example 3.15: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.3}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right\}$. Clearly $\lambda \nsubseteq cl(int(\lambda))$ and $\lambda \subseteq int(cl(\lambda))$. Therefore λ is an intuitionistic fuzzy pre-open set but not an intuitionistic fuzzy semi-open set in X.

Example 3.16: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.45}, \frac{b}{0.35}\right)\right\}$. Clearly $\lambda \subseteq cl(int(\lambda))$ and $\lambda \subseteq int(cl(\lambda))$. Therefore, λ is both an intuitionistic fuzzy semi-open set and an intuitionistic fuzzy pre-open set in X.

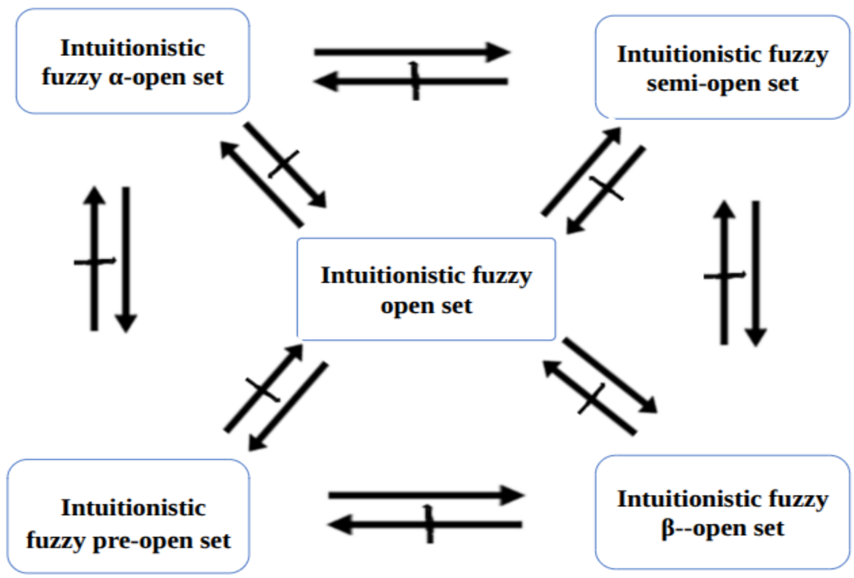
Example 3.17: Let $X = \{a, b\}$ and $I = \{\underline{0}, \underline{1}, A, B\}$ be an intuitionistic fuzzy topological space on X, where the intuitionistic fuzzy sets A and B are defined as $A = \left\{x, \left(\frac{a}{0.2}, \frac{b}{0.7}\right)\right\}$, $B = \left\{x, \left(\frac{a}{0.6}, \frac{b}{0.3}\right)\right\}$. Now, consider an intuitionistic fuzzy set λ in X defined by $\lambda = \left\{x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right\}$.Clearly $\lambda \nsubseteq cl(int(\lambda))$ and $\lambda \nsubseteq int(cl(\lambda))$. Therefore, λ is an neither intuitionistic fuzzy semi-open set nor not an intuitionistic fuzzy pre-open Set in the intuitionistic fuzzy topological space(X, I).

4. CONCLUSION.

In this paper, the concept of intuitionistic fuzzy open sets in intuitionistic fuzzy topological spaces has been studied. Several fundamental properties of these sets were examined and their relationships with other related classes of intuitionistic fuzzy sets were established. The validity of these relationships was illustrated with the help of suitable counterexamples, which are summarized in Figures 1 and 2. The results obtained contribute to a clearer understanding of the

ISBN: 978-93-48954-63-3

structure of intuitionistic fuzzy open sets and may provide a basis for further research in intuitionistic fuzzy topology.



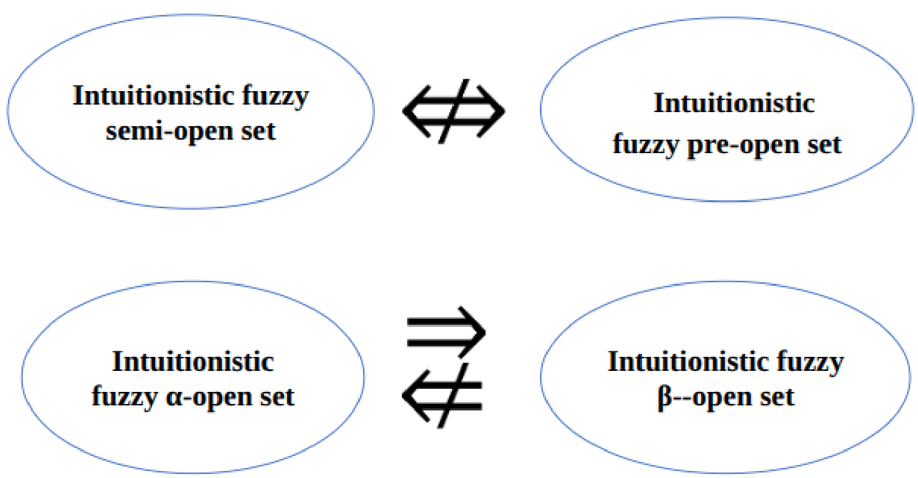


Figure – 2: Relationships of Intuitionistic Fuzzy Open Sets

ISBN: 978-93-48954-63-3

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