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GENERALIZED FORMS OF FUZZY OPEN SETS AND MAPS

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Abstract: In this paper we studied the concept of some fuzzy open sets (GFOS) and continuous map (GFCM) in generalized fuzzy topological space (GFTS) and their characterization by making the use of some supporting examples.

Keywords: Generalized fuzzy sets (GFT), generalized fuzzy topology (GFT), generalized fuzzy open sets (GFOS), and Generalized Fuzzy Continuous map (GFCM).

1. INTRODUCTION

Recently, fuzzy set theory has become very useful in many applications including IT, knowledge-based solutions, risk analysis, control systems, and the like. This theory, however, has now become a rigorous framework for processing the issues faced in complex, multilayered phenomena that conventional methods informed by probability theory and binary logic fall short of handling. Continuity is a fundamental mathematical concept, which causes much generalized concept exploration. In topological spaces, Levine introduced the ideas of semi-open sets and semi-continuity and studied their specific properties [3]. Zadeh developed fuzzy set theory as a natural extension of classical crisp sets [1]. In crisp sets, an element either belongs to or is excluded from the set, but in fuzzy sets an element can have a membership value that comes anywhere from 0 to 1. So, fuzzy set theory is the general approach on which a large number of realistic problems are composed. In the last few decades, much research has been done in fuzzy topology, and it has emerged as a central subject in fuzzy mathematics and has been widely accepted in the theoretical and applied sciences. Since general topology is treated as a special application of fuzzy topology, given that membership values are kept to 0 or 1, the vast majority of research took the classical topological concepts into practice with fuzzy set. Chang used fuzzy sets as foundational concepts in general topology, and also for fuzzy set theory such as fuzzy open sets, closed sets, neighborhoods, interiors, and continuity [2]. These factors greatly aided the development of fuzzy topological structures and their applications. In

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classical or crisp set theory, the element's membership to a set is strictly determined. In fuzzy topological spaces, though, membership becomes gradual rather than absolute. Several generalisation concepts have been adopted by researchers to generalize topology properties extending fuzzy spaces. Azad laid out fuzzy semi-continuity, fuzzy almost continuity, and fuzzy weakly continuity [4]. Bin Shahana subsequently presented two concepts, fuzzy strong semi-continuity and fuzzy pre-continuity [5]. Important properties of fuzzy continuous functions were further investigated in Zhang [6]. As part of further studies, we explored generalized open sets and mappings as well. Mining with the original work of Thakur and Singh led to the discovery for fuzzy semi-pre-open sets and fuzzy semi-pre-continuity [7]. Similar to generalizations, Császár investigated the generalization of open sets within generalized topological spaces and established crucial theoretical roots for further generalizations [8]. Saraf, Navalagi, and Khanna investigated the structure of fuzzy semi-pre-generalized closed sets alongside their behavior in fuzzy topological spaces [9]. Another contribution is that Palani Chetty developed generalized fuzzy topology, developing classical topological principles in a general fuzzy setting [10]. Taken together, these studies illustrate the increased significance of generalized fuzzy open-and-mapped forms in fuzzy topology. As fuzzy mathematical structures continue to grow, and the principles introduced there continue to evolve, new avenues emerge for research and applications of fuzzy mathematical structures in the realms of theory and applications.

2. PRELIMINARIES

Definition 2.1: A crisp set is a well-defined collection of objects those elements of universal set which satisfy a certain defining property are called the members of the set and others are called nonmembers of the set. To a crisp set is associated a function called characteristic function that declares which element of X are the members of the set and which are not. The characteristic function of a set A is given as;

$$\chi_n = \begin{cases} 1 & \text{for x belongs A} \\ 0 & \text{for x not belongs to A} \end{cases}$$

The characteristic function maps elements of X to elements of the pair of two real's {0, 1}, which is formally expressed by $\chi_n: X \rightarrow \{0,1\}$. When $\chi_A(x) = 1$, x is declared to be a member of set A and when $\chi_A(x) = 0$, x is declared to the nonmember of set A.

Definition 2.2: If X is a collection of objects denoted generally by x, then a fuzzy set A in X is a set of ordered pairs

$$A = \{(x, \mu_A(x))/x \in X\}$$

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$\mu_A(x)$is called the membership function or grade of membership (also known as degree of compatibility or degree of truth) of x in A that maps X to the membership space. Let X be a universal set. We consider fuzzy set A, $A: X \rightarrow 1$ defined on X. The collection of all fuzzy sets defined on X is denoted as I^x. Thus

$$I^x = \{A: \text{A is a fuzzy sets}, A: X \rightarrow 1\}$$

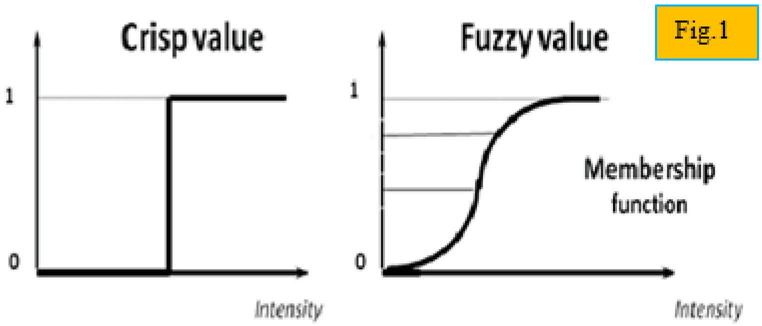

Fig.1

Definition 2.3: Let X be a non-empty crisp set. We take X as the universal set and fuzzy sets in question are defined over X. The collection of fuzzy sets defined on X is denoted as I^x, where $I = [0,1]$ is the range space for all the fuzzy sets under consideration.

A family τ of fuzzy sets on X is called a fuzzy topology on X, it is satisfies the following conditions:

i) The null fuzzy set O and the whole fuzzy set 1 belongs to τ.

ii) Any union of members of τ is also in τ.

iii) Finite intersection of members of τ is in τ.

If τ is a fuzzy topology on X, then the pair (X, τ) is called a fuzzy topological space.

Definition 2.4: Let (X, τ) be a fuzzy topological space, then the members of τ are called fuzzy open sets, while the complement of the members of τ are called fuzzy closed sets.

Remark 2.1: The collection {0,1} containing null fuzzy set 0 and the whole fuzzy set 1 is a fuzzy topology on X and may be called as fuzzy indiscrete topology on X. The collection containing all possible fuzzy sets on X is a fuzzy topology on X This fuzzy topology is called fuzzy discrete topology.

Definition 2.5: Let (X, τ) be a fuzzy topological space and let A be a fuzzy set on X. Then

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the interior of fuzzy set A is denoted by int(A) and is the largest open set containedin A. Let (X, τ) be a fuzzy topological space and let A be a fuzzy set on X, then the interior of A is defined as

$$\mathrm{Int}(A) = \{O;\ O \leq A, O \in \tau\}$$

In other words, interior of a fuzzy set A in a topological space (X, T) is the union of all fuzzy open sets in X contained in A.

Definition 2.6: Let (X, τ) be a fuzzy topological space and let A be a fuzzy set on X. Then the closer of fuzzy set A is denoted by cl(A) and is the smallest closed super set of A. Let (X, τ) be a fuzzy topological space and let A be a fuzzy set on X, then thecloser of A is defined as

$$\mathrm{Cl}(A) = \{C;\ C \geq A, C' \in \tau\}$$

In other words, closer of a fuzzy set A in a topological space (X, τ) is the intersection of all fuzzy closed sets in X containing A.

3. GENERALIZED FUZZY TOPOLOGICAL SPACE (GFTS)

Definition 3.1: Let U be a Universal set. A family T_G of fuzzy sets on U is called a Generalized fuzzy topology (GFT) on U is denoted by I^u, it satisfies the following conditions:

i) $0 \in T_G$

ii) $\cup_{j\in J} \eta_j \in T_G$, for each index $j \in J$

Then the pair (U, T_G) is called a GFTS.

Remark 3.1: (a) The null fuzzy set 0 is a GFT on U and may be called as Generalized fuzzy indiscrete topology (GFIDT) on U (b)The collection containing all possible Generalized fuzzy sets on U is a GFT on U .This Generalized fuzzy topology is called Generalized fuzzy discrete topology (GFDT)

Example 3.1: Let $U = \{u_1, u_2\}$ and $\eta_1, \eta_{2,}\eta_{3,}\eta_4$ belongs to I^u be Generalized fuzzy sets (GFS) defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.8, \chi_{\eta_2}(u_1) = 0.6, \chi_{\eta_2}(u_2) = 0.7$, $\chi_{\eta_3}(u_1) = 0.6, \chi_{\eta_3}(u_2) = 0.8$ and $\chi_{\eta_4}(u_1) = 0.1, \chi_{\eta_4}(u_2) = 0.8$. Then $T_G = \{0,, \eta_1, \eta_{2,}\eta_{3,}\eta_4\}$ is GFTS on U and $0, \eta_1, \eta_{2,}\eta_3$ and η_4 are Generalized fuzzy open sets (GFOS).

Definition 3.2: Let $(\mathbf{U}, \mathbf{T_G})$ be GFTS, the complement of the members of T are called

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generalized fuzzy closed sets (GFCS). In Example 3.1 $\chi_{\eta_1'}(u_1) = 0.7, \chi_{\eta_1'}(u_2) = 0.2$, $\chi_{\eta_2'}(u_1) = 0.4, \chi_{\eta_2'}(u_2) = 0.3$ $\chi_{\eta_3'}(u_1) = 0.4$, $\chi_{\eta_3'}(u_2) = 0.2$ and $\chi_{\eta_4'}(u_1) = 0.9, \chi_{\eta_4'}(u_2) = 0.2$ are Generalized fuzzy closed sets (GFCS)

Definition 3.3: Let (U, T) be a GFTS and let η be a fuzzy set on U. Then the interior of fuzzy set η is denoted by $i_{T_G}(\eta)$and is the largest open set contained in η, $i_{T_G}(\eta)$= {I; I ≤ η, I ∈ T} i.e. interior of a fuzzy set η in GFTS is the union of all GFOS in U contained in η.

Definition 3.4: Let(U, T_G) be a GFTS and let η be a fuzzy set on U. Then the closer of fuzzy set η is denoted by $c_{T_G}(\eta)$ and is the smallest closed super set of η, $c_{T_G}(\eta)$ = {C; C ≥ η, C′ ∈ T} i.e. closer of a fuzzy set η in a GFTS is the intersection of all GFCS in U containing η.

4. GENERALIZED FUZZY OPEN SETS (GFOS)

Definition 4.1: If $(U, T_G)$is a GFTS, then any fuzzy set $\eta \in I^u$ is known as Generalized fuzzy semi-open set (GFSOS) if $\eta \subseteq c_{T_G}(i_{T_G}(\eta))$

Example 4.1: Let $U = \{u_1, u_2\}$ and η_1, η_2, η_3 belongs to I^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.5$, $\chi_{\eta_2}(u_1) = 0.5, \chi_{\eta_2}(u_2) = 0.3$ and $\chi_{\eta_3}(u_1) = 0.5, \chi_{\eta_3}(u_2) = 0.5$. Then $T = \{0, \eta_1, \eta_2, \eta_3\}$ is a GFTS on U, Now let $\chi_\eta(u_1) = 0.4, \chi_\eta(u_2) = 0.4$, then $i_{T_G}(\eta) = 0$ and $c_{T_G}\left(i_{T_G}(\eta)\right) = c_{T_G}(0) = \eta_3$. Therefore, $\eta \subseteq c_{T_G}(i_{T_G}(\eta))$, Thus η is GFSOS.

Theorem 4.1: GFOS⇒ ⇍GFSOS

Proof: Let (U, T_G)be a GFTS and η be any GFOS in U, then $i_{T_G}(\eta) = \eta$, since $\eta \subseteq c_{T_G}(\eta)$.Now we have $\eta = i_{T_G}(\eta) \subseteq c_{T_G}(i_{T_G}(\eta))$, thus $\eta \subseteq c_{T_G}(i_{T_G}(\eta))$, η is GFSOS.

Remark 4.1: The converse of Theorem4.1 is not true in general as in example 4.1 $\chi_\eta(u_1) = 0.4, \chi_\eta(u_2) = 0.4$ is GFSOS, but η is not GFOS.

Definition 4.2: If $(U, T_G)$is be a GFTS, then any fuzzy set η is known as Generalized fuzzy $\alpha -$ open set (GFαOS) if, $\eta \subseteq i_{T_G}(c_{T_G}(i_{T_G}(\eta)))$

Example 4.2: Let $U = \{u_1, u_2, u_3\}$ and η_1, η_2, η_3 belongs to I^u be Generalized fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.2, \chi_{\eta_1}(u_3) = 0.4, \chi_{\eta_2}(u_1) = 0.1, \chi_{\eta_2}(u_2) =$

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$0.2, \chi_{\eta_1}(u_3) = 0.5$and $\chi_{\eta_3}(u_1) = 0.3, \chi_{\eta_3}(u_2) = 0.2, \chi_{\eta_3}(u_3) = 0.5$ Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$ is a FTS on U Now let $\chi_\eta(u_1) = 0.2, \chi_\eta(u_2) = 0.2, \chi_\eta(u_3) = 0.5$, then then $i_{T_G}(\eta) = \eta_2$ and $c_{T_G}\left(i_{T_G}(\eta)\right) = \eta_3'$ $i_T(c_{T_G}\left(i_{T_G}(\eta)\right)) = \eta_3$ Therefore, $\eta \subseteq i_{T_G}(c_{T_G}\left(i_{T_G}(\eta)\right))$, Thus η is GFαOS.

Theorem4.2: GFOS⇒ ⇍GFα OS

Proof: Let (U, T_G)be a GFTS and η be any GFOS in I^u then $i_{T_G}(\eta) = \eta$, since $\eta \subseteq c_{T_G}(\eta)$.we have $\eta = i_{T_G}(\eta) \subseteq c_{T_G}(i_{T_G}(\eta))$,thus $\eta \subseteq c_{T_G}(i_{T_G}(\eta))$, $\eta_1 \subseteq \eta_2$then $i_{T_G}(\eta_1) \subseteq i_{T_G}(\eta_2)$;$i_{T_G}(\eta) \subseteq i_{T_G}(c_{T_G}\left(i_{T_G}(\eta)\right))$ thus, $\eta \subseteq i_{T_G}(c_{T_G}\left(i_{T_G}(\eta)\right))$, η is GFαOS.

Remark4.2: The converse of Theorem4.2 is not true in general as in example 4.2 $\chi_\eta(u_1) = 0.2, \chi_\eta(u_2) = 0.2, \chi_\eta(u_3) = 0.5$ is GFαOS but not open set.

Definition 4.3: If $(U, T_G)$is a GFTS, then any fuzzy set η is known as Generalized fuzzy Pre-open set (GFPOS) if, $\eta \subseteq i_{T_G}(c_{T_G}(\eta))$

Example 4.3: Let $U = \{u_1, u_2,\}$ and η_1, η_2, η_3 belongs toI^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.8$, $\chi_{\eta_2}(u_1) = 0.5, \chi_{\eta_2}(u_2) = 0.4$ and $\chi_{\eta_3}(u_1) = 0.5, \chi_{\eta_3}(u_2) = 0.8$, Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$is a GFTS on U Now let $\chi_\eta(u_1) = 0.4, \chi_\eta(u_2) = 0.4$, then $c_{T_G}(\eta) = \boldsymbol{\eta_2}'$ and $i_{T_G}\left(c_{T_G}(\eta)\right) = \eta_2$. Therefore, $\eta \subseteq i_{T_G}(c_{T_G}(\eta))$Thus η is GFPOS.

Theorem4.3: GFOS⇒ ⇍GFPOS

Proof: Let (U, T_G)be GFTS and η be GFOS in U. then $i_{T_G}(\eta) = \eta$,$c_{T_G}(\eta)$is smallest closed set containing η; $\eta \subseteq c_{T_G}(\eta)$,$i_{T_G}(\eta) \subseteq i_{T_G}(c_{T_G}(\eta))$, $\eta \subseteq i_{T_G}(c_{T_G}(\eta))$, η is GFPOS.

Remark 4.3: The converse of Theorem4.3 is not true in general as in example 4.3 $\chi_\eta(u_1) = 0.4, \chi_\eta(u_2) = 0.4$ is GFPOS, but η is not **G**FOS.

Definition 4.4: If $(U, T_G)$is a GFTS, then any fuzzy set η is known as Generalized fuzzy β – open set (GFβOS) if, $\eta \subseteq c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$

Example 4.4: Let $U = \{u_1, u_2,\}$ and η_1, η_2, η_3 belongs toI^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.8$, $\chi_{\eta_2}(u_1) = 0.5, \chi_{\eta_2}(u_2) = 0.4$ and $\chi_{\eta_3}(u_1) = 0.5, \chi_{\eta_3}(u_2) = 0.8$, Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$is a GFTS on U Now let $\chi_\eta(u_1) = 0.4, \chi_\eta(u_2) = 0.4$, then $c_{T_G}(\eta) = \eta_2'$ and $i_{T_G}\left(c_{T_G}(\eta)\right) = \eta_2$ $c_{T_G}(i_{T_G}\left(c_{T_G}(\eta)\right)) = \eta_2'$. Therefore, $\eta \subseteq$

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$c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$Thus η is GFβOS.

Theorem 4.4: GFOS⇒ ⇍GFβOS

Proof: Let (U, T_G)be a GFTS and η be any FOS in U then $i_{T_G}(\eta) = \eta$, $c_{T_G}(\eta)$is smallest closed set containing η; $\eta \subseteq c_{T_G}(\eta)$,$i_T(\eta) \subseteq i_{T_G}(c_{T_G}(\eta))$, $\eta \subseteq i_{T_G}(c_{T_G}(\eta))$,Apply c_{T_G} on both sides , $c_{T_G}(\eta) \subseteq c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$ as we know $\eta \subseteq c_{T_G}(\eta)$, $\eta \subseteq c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$, η is GFβOS.

Remark 4.4: The converse of Theorem4.4 is not true in general as in example 4.4 $\chi_\eta(u_1) = 0.7, \chi_\eta(u_2) = 0.9$ is GFβOS, but η is not GFOS.

Theorem 4.5: GFαOS⇒ ⇍ GFSOS.

Proof: Let (U, T_G)be GFTS and η be GFαOS in U. then $\eta \subseteq i_{T_G}(c_{T_G}\left(i_{T_G}(\eta)\right))$.as we know $i_{T_G}(\eta) \subseteq \eta$, $i_{T_G}(c_{T_G}\left(i_{T_G}(\eta)\right)) \subseteq c_{T_G}\left(i_{T_G}(\eta)\right)$, $\eta \subseteq i_{T_G}(c_{T_G}\left(i_{T_G}(\eta)\right)) \subseteq c_{T_G}\left(i_{T_G}(\eta)\right)$. Thus, $\eta \subseteq c_{T_G}\left(i_{T_G}(\eta)\right)$. η is GFSOS.

Remark 4.5: The converse of Theorem4.5 is not true in general as; Let $U = \{u_1, u_2, u_3\}$ and η_1, η_2, η_3 belongs to I^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.2, \chi_{\eta_1}(u_3) = 0.4$, $\chi_{\eta_2}(u_1) = 0.1, \chi_{\eta_2}(u_2) = 0.2, \chi_{\eta_2}(u_3) = 0.7$ and ,$\chi_{\eta_3}(u_1) = 0.3, \chi_{\eta_3}(u_2) = 0.2, \chi_{\eta_3}(u_3) = 0.7$. Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$ is a GFTS on U, now let $\chi_\eta(u_1) = 0.2, \chi_\eta(u_2) = 0.5, \chi_\eta(u_3) = 0.3$, Thus η is GFSOS but not GFαOS.

Theorem 4.6: GFPOS⇒ ⇍GFβOS

Proof: Let (U, T_G)be GFTS and η be GFPOS in U. Then $\eta \subseteq i_{T_G}(c_{T_G}(\eta))$,Apply closure on both sides we get, $c_{T_G}(\eta) \subseteq c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$ but as $\eta \subseteq c_{T_G}(\eta)$, $\eta \subseteq c_{T_G}(\eta) \subseteq c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$; $\eta \subseteq c_{T_G}(i_{T_G}(c_{T_G}(\eta)))$,Thus η is GFβOS.

Remark 4.6: The converse of Theorem4.6 is not true in general as; **:** Let $U = \{u_1, u_2\}$ and η_1, η_2, η_3belongs to I^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.1, \chi_{\eta_1}(u_2) = 0.3$, $\chi_{\eta_2}(u_1) = 0.4, \chi_{\eta_2}(u_2) = 0.2$, $\chi_{\eta_3}(u_1) = 0.4, \chi_{\eta_3}(u_2) = 0.3$ and . Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$ is a GFTS on U, now let $\chi_\eta(u_1) = 0.4, \chi_\eta(u_2) = 0.4$, Thus η is GFβOS but not GFPOS.

Remark 4.7: GFSOS and GFPOS are independent to each other (GFSOS ⇎GFPOS) as shown in example.

ISBN: 978-93-48954-63-3

Example 4.5: (a) Let $U = \{u_1, u_2, u_3\}$ and η_1, η_2, η_3 belongs to I^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.3, \chi_{\eta_1}(u_2) = 0.2, \chi_{\eta_1}(u_3) = 0.4,$ $\chi_{\eta_2}(u_1) = 0.1, \chi_{\eta_2}(u_2) = 0.2, \chi_{\eta_2}(u_3) = 0.7$ and $\chi_{\eta_3}(u_1) = 0.3, \chi_{\eta_3}(u_2) = 0.2, \chi_{\eta_3}(u_3) = 0.7$. Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$ is a GFTS on U, Now let $\chi_\eta(u_1) = 0.2, \chi_\eta(u_2) = 0.1, \chi_\eta(u_3) = 0.4$, then $\eta \nsubseteq c_{T_G}\left(i_{T_G}(\eta)\right)$ but $\eta \subseteq i_{T_G}(c_{T_G}(\eta))$. Thus η is GFPOS but not GFSOS.

(b) Let $U = \{u_1, u_2\}$ and η_1, η_2, η_3 belongs to I^u be fuzzy sets defined as $\chi_{\eta_1}(u_1) = 0.1, \chi_{\eta_1}(u_2) = 0.3,$ $\chi_{\eta_2}(u_1) = 0.4, \chi_{\eta_2}(u_2) = 0.2$ and $\chi_{\eta_3}(u_1) = 0.4, \chi_{\eta_3}(u_2) = 0.3$ Then $T_G = \{0, \eta_1, \eta_2, \eta_3\}$ is a FTS on U. Let $\chi_\eta(u_1) = 0.4$, $\chi_\eta(u_2) = 0.4$ then $\eta \nsubseteq i_{T_G}(c_{T_G}(\eta))$but $\eta \subseteq c_{T_G}\left(i_{T_G}(\eta)\right)$. Thus η is GFSOS but not GFPOS.

5. GENERALIZED FUZZY CONTINUOUS MAP (GFCM)

Definition 5.1: Let (U, T_{G_1}) and (V, T_{G_2})be two Generalized fuzzy topological spaces (GFTS) and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then F is called Generalized fuzzy continuous mapping (GFCM) if $F^{-1}(\eta)$ is Generalized fuzzy open (GFOS) in $(U, T_{G_1})$for each Generalized fuzzy open set (GFOS) η in (V, T_{G_2}).

Example 5.1: Let $U = \{u_1, u_2\}$ and $V = \{v_1, v_2\}$. Let $\chi_{\eta_1}(u_1) = \chi_{\eta_4}(v_1) = 0.5, \chi_{\eta_1}(u_2) = \chi_{\eta_4}(v_2) = 0.3,$ $\chi_{\eta_2}(u_1) = \chi_{\eta_5}(v_1) = 0.1, \chi_{\eta_2}(u_2) = \chi_{\eta_5}(v_2) = 0.6,$ $\chi_{\eta_3}(u_1) = \chi_{\eta_6}(v_1) = 0.5, \chi_{\eta_3}(u_2) = \chi_{\eta_6}(v_2) = 0.6,$ Then $T_{G_1} = \{0, \eta_1, \eta_2, \eta_3\}$ and $T_{G_2} = \{0, \eta_4, \eta_5, \eta_6\}$ are GFTS on U and V respectively. Now consider the mapping $F: (U, T_{G_1}) \to (V, T_{G_2})$ defined by $F(u_1) = v_1$ and $F(u_2) = v_2$. We observe that $F^{-1}(0) = 0$, $F^{-1}\left(\chi_{\eta_4}(v_1)\right) = \chi_{\eta_1}(u_1) = 0.5$, $F^{-1}\left(\chi_{\eta_4}(v_2)\right) = \chi_{\eta_1}(u_2) = 0.3$, $F^{-1}\left(\chi_{\eta_5}(v_1)\right) = \chi_{\eta_2}(u_1) = 0.1$, $F^{-1}\left(\chi_{\eta_5}(v_2)\right) = \chi_{\eta_2}(u_2) = 0.6$, $F^{-1}\left(\chi_{\eta_6}(v_1)\right) = \chi_{\eta_3}(u_1) = 0.5$ and $F^{-1}\left(\chi_{\eta_6}(v_2)\right) = \chi_{\eta_3}(u_2) = 0.6$ where $0, \eta_1, \eta_2, \eta_3$ are GFOS in (U, T_{G_1}). Hence $F: (U, T_{G_1}) \to (V, T_{G_2})$ is a generalized fuzzy continuous map (GFCM).

Theorem 5.1: Let (U, T_{G_1}) and (V, T_{G_2})be two GFTS and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then following statements are all equivalent:

a. F is GFCM.
b. $F^{-1}(\eta)$ is fuzzy closed in (U, T_{G_1}) for each Generalized fuzzy closed set(GFCS) η in (V, T_{G_2})

ISBN: 978-93-48954-63-3

Theorem 5.2: Let(U, T_{G_1}) and(V, T_{G_2})be two GFTM and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a map. Let B be a basis for (V, T_{G_2})if for each fuzzy basis open set μ in B, $F^{-1}(\mu))$ is fuzzy open in (U, T_{G_1}), then $F: (U, T_{G_1}) \to (V, T_{G_2})$ is GFCM.

Theorem 5.3: Let(U, T_{G_1}) ,(V, T_{G_2}) and (W, T_3)be GFTM and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ and $G: (V, T_{G_2}) \to (W, T_{G_3})$ be GFCM, then $GoF: (U, T_{G_1}) \to (W, T_{G_3})$ is also GFTM.

Definition 5.2: Let (U, T_{G_1}) and (V, T_{G_2})be two GFTS and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then F is called Generalized fuzzy semi –continuous mapping (GFSCM) if $F^{-1}(\eta)$ is GFSCM in $(U, T_{G_1})$for each GFSCM η in (V, T_{G_2}).

Example 5.2: Let $U = \{u_1, u_2\}$ and $V = \{v_1, v_2\}$. Let $\chi_{\eta_1}(u_1) = \chi_{\eta_4}(v_1) = 0.5, \chi_{\eta_1}(u_2) = \chi_{\eta_4}(v_2) = 0.3$, $\chi_{\eta_2}(u_1) = \chi_{\eta_5}(v_1) = 0.1, \chi_{\eta_2}(u_2) = \chi_{\eta_5}(v_2) = 0.6$, $\chi_{\eta_3}(u_1) = \chi_{\eta_6}(v_1) = 0.5, \chi_{\eta_3}(u_2) = \chi_{\eta_6}(v_2) = 0.6$, and $\chi_{\eta_7}(v_1) = 0.4$, $\chi_{\eta_7}(v_2) = 0.4$,Then $T_{G_1} = \{0, \eta_1, \eta_2, \eta_3\}$ and $T_{G_2} = \{0, \eta_4, \eta_5, \eta_6, \eta_7\}$ are GFTS on U and V respectively. Now consider the mapping $F: (U, T_{G_1}) \to (V, T_{G_2})$ defined by $F(u_1) = v_1$ and $F(u_2) = v_2$. We observe that $F^{-1}(0) = 0$, $F^{-1}\left(\chi_{\eta_4}(v_1)\right) = \chi_{\eta_1}(u_1) = 0.5$, $F^{-1}\left(\chi_{\eta_4}(v_2)\right) = \chi_{\eta_1}(u_2) = 0.3$, $F^{-1}\left(\chi_{\eta_5}(v_1)\right) = \chi_{\eta_2}(u_1) = 0.1$, $F^{-1}\left(\chi_{\eta_5}(v_2)\right) = \chi_{\eta_2}(u_2) = 0.6$, $F^{-1}\left(\chi_{\eta_6}(v_1)\right) = \chi_{\eta_3}(u_1) = 0.5$ and $F^{-1}\left(\chi_{\eta_6}(v_2)\right) = \chi_{\eta_3}(u_2) = 0.6$ $F^{-1}\left(\chi_{\eta_7}(v_1)\right) = 0.4$ and $F^{-1}\left(\chi_{\eta_7}(v_2)\right) = 0.4$ where $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6$ and η_7 are GFSOS in (U, T_{G_1}). Hence $F: (U, T_{G_1}) \to (V, T_{G_2})$ is GFSCM.

Theorem 5.4: GFCM$\Rightarrow \nLeftarrow$ GFSCM

Proof: Let (U, T_{G_1}) and (V, T_{G_2})be two GFTS and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then by definition $F^{-1}(\eta)$ is GFOS in U. for every open set in V. since we know that every GFOS in GFTS is GFSOS. Therefore $F^{-1}(\eta)$ is GFSOS in U. for GFSOS in η in V. Hence $F: (U, T_{G_1}) \to (V, T_{G_2})$ is a GFSCM

Remark 5.1: The converse of Theorem 5.4 is not true.

Example 5.3: In Example 5.2, $F: (U, T_{G_1}) \to (V, T_{G_2})$ is a GFSCM but not GFCM

Definition 5.3: Let (U, T_{G_1}) and (V, T_{G_2})be two GFTS and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then F is called Generalized fuzzy α –continuous mapping (GFα CM) if $F^{-1}(\eta)$ is GFαOS in $(U, T_{G_1})$for each GFαOS η in (V, T_{G_2}).

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Example 5.4: Let $U = \{u_1, u_2, u_3\}$ and $V = \{v_1, v_2, v_3\}$. Let $\chi_{\eta_1}(u_1) = \chi_{\eta_4}(v_1) = 0.3, \chi_{\eta_1}(u_2) = \chi_{\eta_4}(v_2) = 0.2, \chi_{\eta_1}(u_3) = \chi_{\eta_4}(v_3) = 0.4,$ $\chi_{\eta_2}(u_1) = \chi_{\eta_5}(v_1) = 0.1, \chi_{\eta_2}(u_2) = \chi_{\eta_5}(v_2) = 0.2, \chi_{\eta_2}(u_3) = \chi_{\eta_5}(v_3) = 0.5,$ $\chi_{\eta_3}(u_1) = \chi_{\eta_6}(v_1) = 0.3, \chi_{\eta_3}(u_2) = \chi_{\eta_6}(v_2) = 0.2, \chi_{\eta_3}(u_3) = \chi_{\eta_6}(v_3) = 0.5,$ and $\chi_{\eta_7}(v_1) = 0.2$, $\chi_{\eta_7}(v_2) = 0.2, \chi_{\eta_7}(v_3) = 0.5$, Then $T_{G_1} = \{0, \eta_1, \eta_2, \eta_3\}$ and $T_{G_2} = \{0, \eta_4, \eta_5, \eta_6, \eta_7\}$ are GFTS on U and V respectively. Now consider the mapping $F: (U, T_{G_1}) \to (V, T_{G_2})$ defined by $F(u_1) = v_1$ and $F(u_2) = v_2$. We observe that $F^{-1}(0) = 0$, $F^{-1}\left(\chi_{\eta_4}(v_1)\right) = \chi_{\eta_1}(u_1) = 0.3$, $F^{-1}\left(\chi_{\eta_4}(v_2)\right) = \chi_{\eta_1}(u_2) = 0.2$, $F^{-1}\left(\chi_{\eta_4}(v_3)\right) = \chi_{\eta_1}(u_3) = 0.4$, $F^{-1}\left(\chi_{\eta_5}(v_1)\right) = \chi_{\eta_2}(u_1) = 0.1$, $F^{-1}\left(\chi_{\eta_5}(v_2)\right) = \chi_{\eta_2}(u_2) = 0.2$, $F^{-1}\left(\chi_{\eta_5}(v_2)\right) = \chi_{\eta_2}(u_2) = 0.5$, $F^{-1}\left(\chi_{\eta_6}(v_1)\right) = \chi_{\eta_3}(u_1) = 0.3$, $F^{-1}\left(\chi_{\eta_6}(v_2)\right) = \chi_{\eta_3}(u_2) = 0.2$, $F^{-1}\left(\chi_{\eta_6}(v_3)\right) = \chi_{\eta_3}(u_3) = 0.5$ and $F^{-1}\left(\chi_{\eta_7}(v_1)\right) = 0.2$, $F^{-1}\left(\chi_{\eta_7}(v_2)\right) = 0.2$, $F^{-1}\left(\chi_{\eta_7}(v_3)\right) = 0.5$ where $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6$ and η_7 are GFαOS in (U, T_{G_1}). Hence $F: (U, T_{G_1}) \to (V, T_{G_2})$ is GFαCM

Theorem 5.5: GFCM⇒ ⇍GFαCM

Proof: Let (U, T_{G_1}) and (V, T_{G_2})be two GFTS and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then by definition $F^{-1}(\eta)$ is GFOS in U. for every GFOS in V. since we know that every GFOS in GFTS is GFαOS. Therefore $F^{-1}(\eta)$ is GFαOS in U. for every GFαOS in η in V. Hence $F: (U, T_{G_1}) \to (V, T_{G_2})$ is a GFαCM.

Remark 5.2: The converse of Theorem 5.5 is not true.

Example 5.5: In Example 4.4, $F: (U, T_{G_1}) \to (V, T_{G_2})$ is a GFαCM but not GFCM

Definition 5.4: Let (U, T_{G_1}) and (V, T_{G_2})be two GFTS and let $F: (U, T_{G_1}) \to (V, T_{G_2})$ be a mapping from set U to set V. Then F is called GFPCM if $F^{-1}(\eta)$ is GFPOS in $(U, T_1)$for each GFPOS set η in (V, T_{G_2}).

Example 5.6: Let $U = \{u_1, u_2\}$ and $V = \{v_1, v_2\}$. Let $\chi_{\eta_1}(u_1) = \chi_{\eta_4}(v_1) = 0.3, \chi_{\eta_1}(u_2) = \chi_{\eta_4}(v_2) = 0.8$, $\chi_{\eta_2}(u_1) = \chi_{\eta_5}(v_1) = 0.5, \chi_{\eta_2}(u_2) = \chi_{\eta_5}(v_2) = 0.4$, $\chi_{\eta_3}(u_1) = \chi_{\eta_6}(v_1) = 0.5, \chi_{\eta_3}(u_2) = \chi_{\eta_6}(v_2) = 0.8,$ and $\chi_{\eta_7}(v_1) = 0.4$, $\chi_{\eta_7}(v_2) = 0.4$,Then $T_{G_1} = \{0, \eta_1, \eta_2, \eta_3\}$ and $T_{G_2} = \{0, \eta_4, \eta_5, \eta_6, \eta_7\}$ are GFTS on U and V respectively. Now consider the mapping $F: (U, T_{G_1}) \to (V, T_{G_2})$ defined by $F(u_1) = v_1$ and $F(u_2) = v_2$. We

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observe that $F^{-1}(0)=0$, $F^{-1}\left(\chi_{\eta_4}(v_1)\right)=\chi_{\eta_1}(u_1)=0.3$, $F^{-1}\left(\chi_{\eta_4}(v_2)\right)=\chi_{\eta_1}(u_2)=0.8$, $F^{-1}\left(\chi_{\eta_5}(v_1)\right)=\chi_{\eta_2}(u_1)=0.5$, $F^{-1}\left(\chi_{\eta_5}(v_2)\right)=\chi_{\eta_2}(u_2)=0.4$, $F^{-1}\left(\chi_{\eta_6}(v_1)\right)=\chi_{\eta_3}(u_1)=0.5$ and $F^{-1}\left(\chi_{\eta_6}(v_2)\right)=\chi_{\eta_3}(u_2)=0.8$ $F^{-1}\left(\chi_{\eta_7}(v_1)\right)=0.4$ and $F^{-1}\left(\chi_{\eta_7}(v_2)\right)=0.4$ where $\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6$ and η_7 are GFPOS in (U,T_{G_1}). Hence $F:(U,T_{G_1})\to(V,T_{G_2})$ is GFPCM.

Theorem 5.6: GFCM⇒ ⇍GFPCM

Proof: Let(U,T_{G_1}) and (V,T_{G_2})be two GFTS and let $F:(U,T_{G_1})\to(V,T_{G_2})$ be a mapping from set U to set V. Then by definition $F^{-1}(\eta)$ is GFOS in U. for every GFOS set in V. since we know that every GFOS in GFTS is GFPOS. Therefore $F^{-1}(\eta)$ is GPOS in U. for every GPOS in η in V. Hence $F:(U,T_{G_1})\to(V,T_{G_2})$ is a GFPCM.

Remark 5.3: The converse of Theorem 5.6 is not true.

Example 5.7: In Example 5.6, $F:(U,T_{G_1})\to(V,T_{G_2})$ is a GFPCM but not GFCM.

Definition 5.5: Let (U,T_{G_1}) and (V,T_{G_2})be two GFTS and let $F:(U,T_{G_1})\to(V,T_{G_2})$ be a mapping from set U to set V. Then F is called GFβCM if $F^{-1}(\eta)$ is GFβOS in $(U,T_{G_1})$for each GFβOS η in (V,T_{G_2}).

Example 5.8: Refer example 5.6

Theorem 5.7: GFCM⇒ ⇍GFβCM

Proof: Let (U,T_{G_1}) and (V,T_{G_2})be two GFTS and let $F:(U,T_{G_1})\to(V,T_{G_2})$ be a mapping from set U to set V. Then by definition $F^{-1}(\eta)$ is GFOS in U. for every GFOS in V. since we know that every GFOS in GFTS is GFβOS. Therefore $F^{-1}(\eta)$ is GFβOSin U, for every GFβOS.in η in V. Hence $F:(U,T_{G_1})\to(V,T_{G_2})$ is a GFβCM.

Remark 5.4: The converse of Theorem 5.7 is not true.

Example 5.9: In Example 5.8, $F:(U,T_{G_1})\to(V,T_{G_2})$ is a GFβCMmap but not GFCM.

Theorem 5.8: GFαCM⇒ ⇍GFSCM.

Proof: Let (U,T_{G_1}) and (V,T_{G_2})be two GFTS and let $F:(U,T_{G_1})\to(V,T_{G_2})$ be a GFαCM from U to V. Then by definition $F^{-1}(\eta)$ is GFαOS in U for every GFOS η in V. Since we

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know that every GFαOS in GFTS is GFSOS. Therefore $F^{-1}(\eta)$ is GFSOS in U for every GFOS η in V. Hence $F:(U,T_{G_1}) \to (V,T_{G_2})$ is a GFSCM.

Remark 5.5: The converse of Theorem 5.8 is not true.

Example 5.10: In Remark 4.5, $F:(U,T_{G_1}) \to (V,T_{G_2})$ is a GFSCM but not GFαCM.

Theorem 5.9: GFPCM⇒ ⇍GFβCM.

Proof: Let (U,T_{G_1}) and (V,T_{G_2})be two GFTS and let $F:(U,T_{G_1}) \to (V,T_{G_2})$ be a GFPCM from U to V. Then by definition $F^{-1}(\eta)$ is GFPOS in U for every GFOS η in V. Since we know that every GFPOS in GFTS is GβOS. Therefore $F^{-1}(\eta)$ is GFβOS in U for every GFOS η in V. Hence $F:(U,T_{G_1}) \to (V,T_{G_2})$ is a GFβCM.

Remark 5.6: The converse of Theorem 5.9 is not true.

Example 5.11: In Remark 4.6 $F:(U,T_{G_1}) \to (V,T_{G_2})$ is a GFβCM but not GFPCM.

Remark 5.7: GFSCM and GFPCM are independent to each other (GFSCM ⇎ GFPCM) as shown in example 4.5(a) and 4.5(b).

6. CONCLUSION

Joining the concept of GFOS to the concept of GFSOS, GFαOS, GF*P*OS, GFβOS on an underlying FTS has some effect as shown in the paper, by making the use of some supporting examples and relation between them, which is shown in following figure-1, Also studied the concept of Continuity of mappings of fuzzy topological spaces and continuity in fuzzy topological spaces, Joining the concept of GFCM to the concept of GFSCM, GFαCM, GFPCM, GFβCM on an underlying GFTS as shown in figure-2

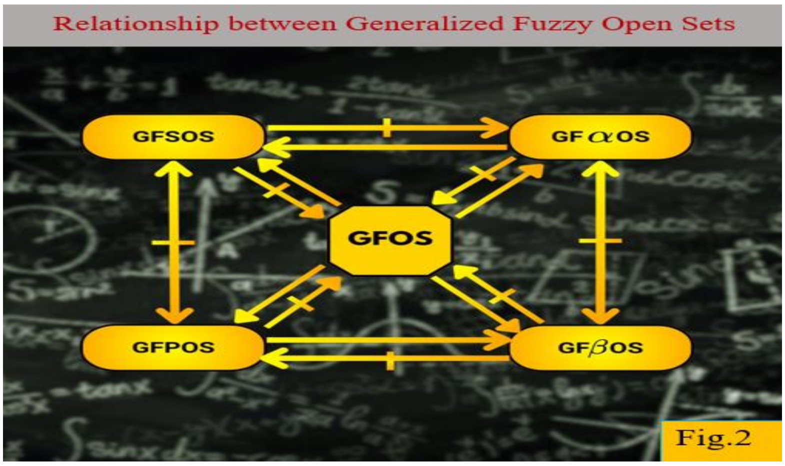

Fig.2

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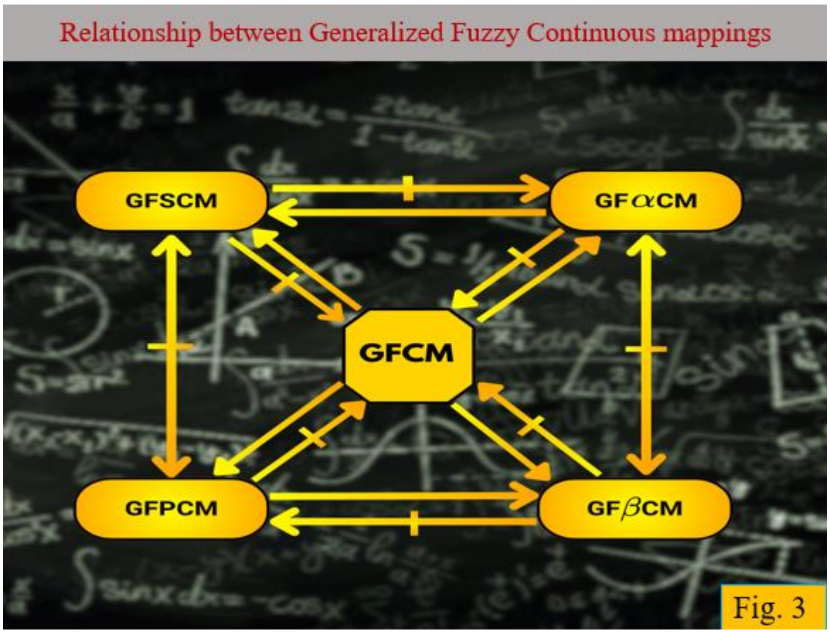

Fig. 3

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