Models of Relativistic Fluid Spheres using class one condition

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ABSTRACT

In the present article we investigate a new exact solution of Einstein field equations for anisotropic compact stars by assuming a new metric potential e^{λ} satisfying the Karmarkar condition of embedded class I space-time. We analyse several physical properties like pressure, density, gravitational red shift, mass function, anisotropy etc. for compact fluid spheres reported in the paper. The new solution is well behaved in all aspects, free from any singularities, satisfies the causality condition and hydrostatic equilibrium condition is well maintained by our models. We also check the stability via Herrera's cracking method. The necessary stability criteria i.e. strong and weak energy conditions, Buchdhal condition, Adiabatic index condition all stand true for our models. We frame our solution for modelling of compact stars PSRJ 1614-2230, 4U1608-52, Cen X-3. The obtained masses and radii are in close agreement with observational data. We interpret compact star models using analytical expressions as well as with the help of graphical representations.

Keywords: Anisotropy; Compact Star; Embedding Class I; Exact Solution; Karmarkar Condition; Metric potential.

1. Introduction

Whenever any star runs out of its nuclear fuel, the outward radiation pressure due to nuclear fusion cannot resist gravitational force and the star collapses under its own gravitational force. The tremendous pressure during collapse becomes energetically favourable for electrons to merge with protons to form neutrons and neutrinos. All these processes are associated with supernova explosion of type II, in which outer layers are blown away with enormous pressure and energy outshining many galaxies with possible leftover cores as neutron stars, strange stars and quark stars. The composition and nature of compact stars has been an active area of research for astrophysicists for the last few decades. By direct observational data it is a challenge to measure parameters like internal composition, mass, radii etc. The investigations of optical and radio astronomers in last few years have unfolded many interesting facts pertaining to compact stars. The developments in observational techniques and theoretical advancements regarding compact objects have persuaded the researchers to investigate deeply inside the internal composition of these stellar objects. A relativistic stellar model can predict many characteristics and features of compact stars. Einstein's theory of general relativity helps to understand the nature and behaviour of various parameters of compact stars. In 1916 Schwarzschild obtained the first exact solution of Einstein field equations for interior of compact stars by considering matter distribution with uniform density. In context of Einstein field equations Oppenheimer and Volkoff, Tolman [1, 2] developed the simplest models of relativistic stellar structures. Later on the physicists got motivated towards contemplating more realistic models of compact star by including the stringent conditions like charge, rotation, anisotropy etc. Compact objects have smaller radii and strong surface gravity and the interior structure have extremely high density. Ruderman's [3] theoretical investigation shows that at very high density of the order 10^{15} gm/cm^3, pressure inside a compact object becomes anisotropic in nature. The pressure has two components radial and transverse [4] and their difference is called anisotropic factor, which describes the interior matter distribution of star. The existence of magnetic field, rotational motion, presence of a solid core, presence of type 3A fluid or of type P superfluid [5] are the factors due to which anisotropy arises. The other factors for anisotropy are Pion condensation and different kinds of phase transition etc [6]. Bowers and Liang [7] theoretically investigated the influence of anisotropy on the characteristics of compact stars. According to Mak and Harko [8] & Sharma and Maharaj [9], anisotropy plays a significant role in the study of dense nuclear matters. Maurya [10] argued that the interaction between particles in compact stars is highly relativistic which results into non uniform distribution of matter throughout the stellar interior. Hence one of the possible reasons for the anisotropy in compact stars could be originate due to the relativistic nature of particles. In a self gravitating system Herrera and Santos [11] studied the possible reasons of local anisotropy and various aspects of the anisotropy. They observed that corresponding to local isotropy, anisotropy causes the configuration more or less stable, cause changes into mass and surface red shift. The work of Dev and Gleiser [12, 13] showed that the physical properties, mass and structure of compact stars are influenced by pressure anisotropy. Mak and Harko [14] worked on the role of anisotropy with linear equation of state. By using suitable metric potential function Maurya et al. [15, 16] investigated an anisotropic solution of Einstein field equations. In context of general relativity Cosenza et al. [17] presented anisotropic models of compact stellar structures. At high density Heintzmann and Hillebrandt [18] studied relativistic, anisotropic neutron star model. Chan et al. [19] proposed that the stability of self gravitating system gets affected due to local anisotropy. Work of Herrera et al. [20] propounded the influence of local anisotropy on the structure and properties of compact objects. For charged anisotropic matter Manuel Malaver [21] generated a new solution for Einstein Maxwell field equation by using barotropic equation of state. By considering quadratic equation of state as proposed by Feroze and Siddiqui [22], Malaver also studied the nature of compact objects, in which matter distribution is anisotropic. With the help of Karmarkar [23] condition, K. N. Singh et al. [24, 25, 26], K. N. Singh and Pant [27], Maurya et al. [28] derived the exact solutions of Einstein's field equations to describe compact star characteristics.

In this paper we work on some new relativistic anisotropic compact star models. We choose a new metric potenial e^{λ} and the imposition of Karmarkar condition leads to the generation of another metric potential e^{ν}. As the solution of our models has positive and finite values of central density and central pressure as well as the metric potentials e^{λ} and e^{ν} got non zero positive values, therefore our solution is free from any physical and geometrical singularities. The outline of our present paper is organized in 7 sections as follows: In section 2 the field equations have been solved using the Karmarkar condition. Section 3 contains a new solution generated for anisotropic compact star models of embedded class I. In section 4, by using boundary conditions values of constants are obtained. Physical analysis of the new solution is discussed in section 5. In Section 6 we analyse the stability of models. In section 7 we summarize all obtained results with suitable discussions.

2. Relativistic Field Equations For Spherically Symmetric Fluid Spheres

The interior of a static spherically symmetric compact object in canonical coordinate $(x^{\kappa}) \equiv (t, r, \theta, \phi)$ is described as

$$ds^2 = -\, r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) - e^{\lambda(r)}dr^2 + e^{\nu(r)}dt^2 \qquad (1)$$

Here ν and λ are function of radial coordinate *r*.

When the matter within the star is anisotropic the energy momentum tensor may be expressed as

$$T_{\kappa\tau} = p_t\left(v_\kappa v_\tau - \eta_\kappa \eta_\tau - \gamma_{\kappa\tau}\right) + p_r \eta_\kappa \eta_\tau + \rho v_\kappa v_\tau \qquad (2)$$

Here ρ is matter density, p_r is the radial pressure and p_t is transverse pressure of the fluid, v_κ is the four velocity and η_κ is the unit space like vector in the direction of radial vector.

The Einstein field equations for the line element (1) and the matter distribution (2) are given by

$$8\pi\rho = \frac{1-e^{-\lambda}}{r^2} + \frac{\lambda' e^{-\lambda}}{r} \qquad (3)$$

$$8\pi p_r = \frac{\nu' e^{-\lambda}}{r} - \frac{1-e^{-\lambda}}{r^2} \qquad (4)$$

$$8\pi p_t = e^{-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu'}{2r} - \frac{\lambda'}{2r}\right) \qquad (5)$$

From equation (4) and (5) we obtain the anisotropy parameter

$$\Delta = p_t - p_r = \frac{e^{-\lambda}}{8\pi}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{\nu'+\lambda'}{2r} + \frac{e^{\lambda}-1}{r^2}\right) \qquad (6)$$

2.1. The Class One Condition

A symmetric tensor $h_{\kappa\tau}$ of 4 dimensional Riemanian space which satisfies the Gauss and Codazzi condition [29] given as

$$R_{\kappa\tau ij} = \varepsilon\left(h_{\kappa i} h_{\tau j} - h_{\kappa j} \Box_{\tau i}\right) \qquad (7)$$

$$h_{\kappa\tau;i} - h_{\kappa i;\tau} = 0 \qquad (8)$$

can be embedded in 5 dimensional Pseudo-Euclidean space. Here ε = +1, when the normal to the manifold is space like or ε = -1, when the normal to the manifold is time like. The symbol (;) represents covariant derivative. For the line element represented in equation (1), the components of Riemann curvature tensor are expressed as below (Newton Singh et al. [30])

$$R_{1212} = \frac{1}{2} r\lambda' \qquad (9)$$

$$R_{1334} = R_{1224}\sin^2\theta = 0 \qquad (10)$$

$$R_{1414} = -e^{\nu}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4}\right) \qquad (11)$$

$$R_{2323} = -e^{\lambda} r^2 \sin^2\theta\left(e^{\lambda} - 1\right) \qquad (12)$$

$$R_{3434} = -\frac{1}{2} r \sin^2\theta\, \nu' e^{\nu-\lambda} \qquad (13)$$

With the non zero components of $h_{\kappa\tau}$, equation (8) reduces to Karmarkar condition which is

$$R_{1414} = \frac{R_{1212}\, R_{3434} + R_{1224}\, R_{1334}}{R_{2323}} \qquad (14)$$

Substituting the corresponding values from equations (9) - (13) in equation (14) we obtain following differential equation

$$\frac{\lambda' e^{\lambda}}{e^{\lambda}-1} = \nu' + \frac{2\nu''}{\nu'} \qquad (15)$$

On integrating the above equation the relation between ν and λ as following

$$e^{\nu} = \left(\delta + \beta \int \sqrt{e^{\lambda}-1}\,dr\right)^2 \qquad (16)$$

Where, δ and β are constants of integration.

With the help of equation (16), equation (6) can be written as

$$\Delta = \frac{\nu'}{4e^{\lambda}}\left(\frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1}\right)\left(\frac{\nu' e^{\nu}}{2r\beta^2} - 1\right) \qquad (17)$$

Where, Δ is anisotropic factor which could be positive or negative. Positive anisotropy factor produces outward force whereas negative anisotropic factor produces inward force. The anisotropy Δ vanishes at the centre ($r = 0$) of the stellar structure.

3. A New Interior Space-Time Metric For Anisotropic Stellar Configurations

To solve equation (16) we assume a metric potential as

$$e^{\lambda} = 1 + ar^2\{1 + \cos(br^2 + c)\}^2 \qquad (18)$$

Where a, b and c are constants, and a & b both have the dimension of length^{-2}.

Substituting the value of e^{λ} from equation (18) in equation (16) we have

$$e^{\nu} = \left[\delta + \frac{\beta\sqrt{a}}{2b}\{br^2 + \sin(br^2 + c)\}\right]^2 \qquad (19)$$

By using equation (18) and (19) we can express the physical parameters ρ (density), p_r (radial pressure), Δ (anisotropic factor) and p_t (transverse pressure) as

$$8\pi\rho = \left[\frac{aU(r)}{1+ar^2\{U(r)\}^2}\right]\left[U(r) + \frac{2[U(r) - br^2\sin(br^2+c)]}{1+ar^2\{U(r)\}^2}\right] \qquad (20)$$

$$8\pi p_r = \frac{U(r)}{[1+ar^2\{U(r)\}^2]}\left[\frac{2\beta\sqrt{a}}{\left[\delta + \frac{\beta\sqrt{a}}{2b}\{br^2+\sin(br^2+c)\}\right]} - a\{U(r)\}\right] \qquad (21)$$

$$\Delta = \frac{ar^2\,U(r)[a\{U(r)\}^3 + 2b\sin(br^2+c)]}{[1+ar^2\{U(r)\}^2]^2} - \left[\frac{\beta\sqrt{a}r^2}{[1+ar^2\{U(r)\}^2]\left[\delta + \frac{\beta\sqrt{a}}{2b}\{br^2+\sin(br^2+c)\}\right]}\right]\left[2b\sin(br^2+c) + \frac{a\{U(r)\}^2[U(r) - 2br^2\sin(br^2+c)]}{\{1+ar^2\{U(r)\}^2\}}\right] \qquad (22)$$

$$8\pi p_t = 8\pi p_r + \Delta \qquad (23)$$

Here

$$U(r) = 1 + \cos(br^2 + c) \qquad (24)$$

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The gradients of radial pressure, anisotropy, transverse pressure and density are expressed as

$$8\pi\frac{dp_r}{dr}=\left[\frac{2ar\{U(r)\}^2[U(r)-2br^2\sin(br^2+c)]}{e^{2\lambda}}\right]\left[aU(r)-\frac{2\beta\sqrt{a}}{e^{\frac{\nu}{2}}}\right]+\left[\frac{2b\,r\sin(br^2+c)}{e^{\lambda}}\left\{aU(r)-\frac{\beta\sqrt{a}}{e^{\frac{\nu}{2}}}\right\}\right]-\left[\frac{2\beta^2ar\{U(r)\}^2}{e^{\lambda}e^{\nu}}\right] \quad (25)$$

$$\frac{d\Delta}{dr}=\left[\frac{2a^2r\{U(r)\}^4+4ab^2r^3\{\cos 2(br^2+c)+\cos(br^2+c)\}+2abr\{2\sin(br^2+c)+\sin 2(br^2+c)\}}{e^{2\lambda}}\right]$$
$$-\left[\frac{4a^2r^3\{U(r)\}^2[U(r)-2br^2\sin(br^2+c)][a\{U(r)\}^3+2b\sin(br^2+c)]}{e^{3\lambda}}\right]$$
$$-\left[\frac{\beta\sqrt{a}[4b^2r^3\cos(br^2+c)+4br\sin(br^2+c)+2ar\{U(r)\}^2\{U(r)-3br^2\sin(br^2+c)\}]}{e^{2\lambda}e^{\frac{\nu}{2}}}\right]$$
$$-\left[\frac{\beta\sqrt{a}[2br^2\sin(br^2+c)+ar^2\{U(r)\}^3]}{2e^{2\lambda}e^{\frac{\nu}{2}}}\right]\left[\frac{8arU(r)\{U(r)-2br^2\sin(br^2+c)\}}{e^{\lambda}}+\frac{2\beta\sqrt{a}rU(r)}{e^{\frac{\nu}{2}}}\right] \quad (26)$$

$$8\pi\frac{dp_t}{dr}=8\pi\frac{dp_r}{dr}+\frac{d\Delta}{dr} \quad (27)$$

And $$8\pi\frac{d\rho}{dr}=\left[\frac{2arU(r)}{e^{3\lambda}}\left[(6e^{\lambda}-8)b\sin(br^2+c)-a\{U(r)\}^3(e^{\lambda}+4)\right]\right]$$
$$-\frac{4ab}{e^{2\lambda}}\left[2rU(r)\sin(br^2+c)+2br^2\{rU(r)-(r+1)\sin^2(br^2+c)\}\right]$$
$$+\frac{1}{e^{3\lambda}}\left[16abr\sin(br^2+c)(e^{\lambda}-1)\{U(r)-2br^2\sin(br^2+c)\}\right] \quad (28)$$

Herrera, Ospino and Prisco [31] introduced a method by which all static spherically symmetric perfect fluid solutions can be obtained by two generating functions, and these two generating functions ψ (r) and Π (r) are given as

$$e^{\nu(r)}=e^{\int\left\{2\psi(r)-\frac{2}{r}\right\}dr} \quad (29)$$

and

$$\Pi(r)=8\pi(p_r-p_t) \quad (30)$$

For our present model these two generating functions are obtained as

$$\psi(r)=\frac{\beta\sqrt{a}r\{1+\cos(br^2+c)\}}{\left[\delta+\frac{\beta\sqrt{a}}{2b}\{br^2+\sin(br^2+c)\}\right]}+\frac{1}{r} \quad (31)$$

$$\Pi(r)=\left[\frac{\beta\sqrt{a}r^2}{[1+ar^2\{U(r)\}^2]\left[\delta+\frac{\beta\sqrt{a}}{2b}\{br^2+\sin(br^2+c)\}\right]}\right]\left[2b\sin(br^2+c)+\frac{a\{U(r)\}^2[U(r)-2br^2\sin(br^2+c)]}{\{1+ar^2\{U(r)\}^2\}}\right]-\frac{ar^2U(r)[a\{U(r)\}^3+2b\sin(br^2+c)]}{[1+ar^2\{U(r)\}^2]^2} \quad (32)$$

Where

$$U(r)=1+\cos(br^2+c)$$

4. Conditions For Continuity Of Interior And Exterior Space-Time Metrics At The Boundary

The values of constants a, δ and β of our model can be find out by matching interior space time to the exterior Schwarzschild solution given by

$$ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 - \sin^2\theta\, d\phi^2) \qquad (33)$$

Where, M is the total mass of the compact star.

Using the condition of continuity of the metric coefficients e^{λ} and e^{ν} at the boundary ($r = r_b$) of compact star and vanishing value of radial pressure (p_r) at the boundary, we obtain

$$e^{-\lambda_b} = 1 - \frac{2M}{r_b} = [1 + ar_b^2\{1+\cos(br_b^2+c)\}^2]^{-1} \qquad (34)$$

$$e^{\nu_b} = 1 - \frac{2M}{r_b} = \left[\delta + \frac{\beta\sqrt{a}}{2b}\{br_b^2 + \sin(br_b^2+c)\}\right]^2 \qquad (35)$$

By using boundary conditions as given by equations (34) and (35) and the radial pressure (p_r) to be zero at $r = r_b$, from equation (21), we obtain

$$\beta = \frac{2\sqrt{a}b\delta\{1+\cos(br_b^2+c)\}}{4b - [a\{1+\cos(br_b^2+c)\}\{br_b^2+\sin(br_b^2+c)\}]} \qquad (36)$$

$$\delta = \frac{4b - [a\{1+\cos(br_b^2+c)\}\{br_b^2+\sin(br_b^2+c)\}]}{4b\left[1+ar_b^2\{1+\cos(br_b^2+c)\}^2\right]^{1/2}} \qquad (37)$$

$$a = \left[\frac{1}{r_b^2\{1+\cos(br_b^2+c)\}^2}\right]\left[\frac{1}{1-\frac{2M}{r_b}} - 1\right] \qquad (38)$$

Constant *a*, δ and β can be determined from equation (36) to (38) with the help of free parameters *b* and *c* for a stellar structure of given mass *M* and radius *R* (or r_b).

The Values of *a*, *b*, *c*, δ, β, radius (*R*), mass (*M*) for the stars PSR J1614-2230, 4U1608-52 and Cen X-3 are obtained by using the boundary conditions and are given in Table 1.

Table 1: The optimum values of the parameters for modelling of three compact stellar objects.

Compact Object	*a* (km^{-2})	*b* (km^{-2})	*c*	δ	β	Computed value of *R* (km)	Computed value of $M/M_\odot$	*u=2M/R*
PSRJ1614-2230	0.001913	0.0038	6.278	0.640483	0.032757	9.69	1.97	0.407
4U1608-52	0.001791	0.00365	6.28	0.674811	0.032747	9.3	1.74	0.374
Cen X-3	0.016845	0.0008	4.215	2.943787	0.031084	9.17	1.49	0.325

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5. Physical Characteristics Of Relativistic Stellar Models

The value of radial pressure (p_r) and transverse pressure (p_t) are equal at the centre and are positive and finite inside the compact stellar object. Therefore

$$8\pi p_{rc} = 8\pi p_{tc} = \frac{2\beta\sqrt{a}(1+\cos c)}{\delta+\frac{\beta\sqrt{a}}{2b}\sin c} - a(1+\cos c)^2 > 0 \tag{39}$$

And the density at the centre

$$8\pi\rho_c = 3\, a(1+\cos c)^2 > 0; \qquad \forall\, a > 0 \tag{40}$$

Also at the interior of the star to satisfy the Zeldovich's condition [32], p_r/ρ at the centre must be ≤ 1.

Hence we have

$$\frac{\delta}{\beta} \geq \frac{b - a\sin c(1+\cos c)}{2\sqrt{a}b(1+\cos c)} \tag{41}$$

By using equation (39) we have

$$\frac{\delta}{\beta} < \frac{4b - a\sin c(1+\cos c)}{2\sqrt{a}b(1+\cos c)} \tag{42}$$

On combining equations (41) and (42) we get a constraint on δ and β as

$$\frac{b - a\sin c(1+\cos c)}{2\sqrt{a}b(1+\cos c)} \leq \frac{\delta}{\beta} < \frac{4b - a\sin c(1+\cos c)}{2\sqrt{a}b(1+\cos c)} \tag{43}$$

By observing the profiles of various parameters such as metric potentials, pressure (radial and transverse), density, gradients and anisotropy with respect to radial coordinate (r/r_b), the physical properties of our models can be analyzed. The metric potentials are continuous and having non-zero positive values inside the star as shown in Fig. 1. The radial and transverse pressure having same value at the centre of the stellar structure and are monotonically decreasing function of r from centre to the surface, and radial pressure becomes zero at the boundary of compact stars (Fig. 2). The profile of matter density is plotted in Fig. 3, which is monotonic decreasing function of radial coordinate r. The ratio of pressure to density is positive and is less than 1, (Fig. 4) which implies the non-exotic nature of fluid distribution. Fig. 5 represents the profile of density gradient and pressure gradient, and these gradients possess negative values everywhere inside the stellar structure which validates the decreasing behaviour of pressure and density in moving from centre to surface. Anisotropic factor is zero at the centre and increases monotonically from the centre to the surface of star, which has been shown in Fig. 6.

5.1. Mass Function And Compactification Parameter

Using the value of metric potential

$$e^{-\lambda} = 1 - \frac{2m}{r} \tag{44}$$

and equation (3), we can obtain the expression for mass function *m(r)* as

$$m(r) = \int_0^r 4\pi\rho r^2 dr = \frac{ar^3\{1+\cos(br^2+c)\}^2}{2[1+ar^2\{1+\cos(br^2+c)\}^2]} \tag{45}$$

The compactness factor for stellar model is

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$$u(r) = \frac{2m(r)}{r} = \frac{ar^2\{1+\cos(br^2+c)\}^2}{[1+ar^2\{1+\cos(br^2+c)\}^2]} \quad (46)$$

Variation of *m(r)* with *r* is displayed in Fig. 7, which shows that mass is regular inside the star and vanishes at centre. Fig. 8 shows compactness factor is monotonically increasing from centre to the boundary of star.

5.2. The Gravitational Red Shift

The gravitation red shift is related with the metric potential e^{ν} as

$$z(r) = e^{-\nu/2} - 1 = \left[\delta + \frac{\beta\sqrt{a}}{2b}\{\{br^2 + \sin(br^2 + c)\}\}\right]^{-1} - 1 \quad (47)$$

The profile of gravitational red shift is given in Fig. 9, which has a monotonic decreasing nature with radial coordinate *r*.

5.3. Analysis Of Energy Conditions

For physically possible configurations the following energy conditions must be satisfied.

Null energy condition (NEC: $\rho(r) \geq 0$),

Weak energy condition (WEC: $\rho(r) - p_r(r) \geq 0$ and $\rho(r) - p_t(r) \geq 0$),

and the Strong energy condition (SEC: $\rho(r) - p_r(r) - 2p_t(r) \geq 0$).

These inequalities are graphically represented in Fig. (10), and our models satisfy the energy conditions for compact stars PSR J1614-2230, 4U1608-52 and Cen X-3.

6. Stability Analysis Of Stellar Models

6.1. Causality Condition

For physically acceptable stellar model the causality condition holds good when both the radial and transverse velocity of sound are less than 1.

The square of radial and transverse velocity of sound within the relativistic fluid sphere can be obtained as

$$v_r^2 = \frac{dp_r}{d\rho} = \frac{dp_r/dr}{d\rho/dr} \quad (48)$$

$$v_t^2 = \frac{dp_t}{d\rho} = \frac{dp_t/dr}{d\rho/dr} \quad (49)$$

In Fig. 11 we observe that both v_r^2 and v_t^2 have values less than 1 and monotonic decreasing function of r everywhere within the star, which satisfy the causality condition.

For a potentially stable anisotropic fluid distribution, transverse velocity (v_t^2) of sound must be less than radial velocity (v_r^2) of sound ($-1 \leq v_t^2 - v_r^2 < 0$). Profile of Fig. 12 shows the potentially stable configuration.

6.2. Relativistic Adiabatic Index

The static equilibrium of a relativistic anisotropic sphere depends on the adiabatic index (the ratio of two specific heats). For neutral equilibrium its value is equal to 4/3, and according to Heintzmann and Hillebrandt's

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[18], for static equilibrium the adiabatic index of the compact star must be greater than 4/3. The adiabatic index is

$$\Gamma = \frac{\rho + p_r}{p_r}\left(\frac{dp_r}{d\rho}\right) \quad (50)$$

Variations in Γ are drawn in Fig. 13, which shows that it is greater than 4/3 everywhere inside the star, hence our model is stable.

6.3. Equilibrium Under Three Different Forces

The stability of a compact star, under three forces namely gravitational force, hydrostatic force and anisotropic force, can be examine by using the generalized Tolman- Oppenheimer- Volkoff (TOV) equation.

$$-\frac{M_g(\rho+p_r)}{r^2}e^{(\lambda-\nu)/2} - \frac{dp_r}{d\rho} + \frac{2\Delta}{r} = 0 \quad (51)$$

Where $M_g(r)$ is gravitational mass inside the star of radius r, and in terms of metric potentials can be expressed as

$$M_g(r) = \frac{1}{2}r^2\nu' e^{(\nu-\lambda)/2} \quad (52)$$

Now equation (51) becomes

$$-\frac{1}{2}\nu'(\rho + p_r) - \frac{dp_r}{d\rho} + \frac{2\Delta}{r} = 0 \quad (53)$$

Equation (53) can be realized in terms of balanced force equation due to gravitational force (F_g), hydrostatic force (F_h) and anisotropic force (F_a) respectively i.e.

$$F_g + F_h + F_a = 0 \quad (54)$$

Where

$$F_g = -\frac{1}{2}\nu'(\rho + p_r) \quad (55)$$

$$F_h = -\frac{dp_r}{d\rho} \quad (56)$$

$$F_a = \frac{2}{r}(p_t - p_r) = \frac{2\Delta}{r} \quad (57)$$

Fig. 14 shows that the combine effect of hydrostatic force (F_h) and anisotropic force (F_a) is counterbalanced by the gravitational force (F_g), which is dominating in nature and keep the system in equilibrium.

7. Results And Conclusion

In this paper, we found a new solution for anisotropic compact stellar objects of embedding class I. We observed that physical parameters, metric potentials ($e^{-\lambda}$), pressure (p, p_t), density (ρ), ratio of radial pressure to density (p_r/ρ), gravitational red shift (z) and velocity of sound (v_r^2, v_r^2) have positive values at the centre and are decreasing monotonically from centre to the boundary as shown in Fig. (1, 2, 3, 4, 9, 11), which provide sufficient conditions for a well behaved solution. The radial pressure (p_r) is finite, positive and decreasing radially outward and becomes zero at the surface of the star whereas transverse pressure has a non zero positive value at the surface, which is expected for a well behaved model. The metric potential (e^{ν}) has finite value at the centre and monotonically decreasing nature from centre to the surface (Fig. 1) which is a necessary condition for physically possible configurations. The interior fluid distribution is non exotic in nature because the values of parameters $\frac{p_r}{\rho}$ and $\frac{p_t}{\rho}$ lies between zero and one as shown in Fig. 4. The anisotropy factor (Δ) and adiabatic index (Γ) should increase radially outward for physically realizable structure which is followed by our model, Fig. (6, 13). The plots of gradients of density $\left(\frac{d\rho}{dr}\right)$, radial pressure $\left(\frac{dp_r}{dr}\right)$ and transverse pressure $\left(\frac{dp_t}{dr}\right)$ are negative which again verify that density, radial pressure and transverse pressure are monotonically decreasing function of *r* (Fig. 5). The variation in mass with radial coordinate r is plotted in Fig. 7, which shows that it is positive, continuous and increasing monotonically from the centre of the star to its surface. For all three stars PSR J1614-2230, 4U1608-52 and Cen X-3 the compactness factor lies within the Buchdahl [33] limit $\left(u \leq \frac{8}{9}\right)$, and increases radially outward, Fig. 8. The red shift is monotonically decreasing readily outward and having maximum value at the centre, Fig. 9. The values of v_r^2 and v_t^2 are everywhere less than one throughout the interior of fluid sphere, therefore causality condition holds good inside the compact objects, Fig. 11. Our solution also shows that the system is in hydro-static equilibrium under the action of gravitational, anisotropic and hydrostatic forces Fig. 14. The dominating gravitational force is counterbalanced by combined effects of anisotropic and hydrostatic forces. The three energy conditions, which are null energy condition (NEC : $\rho(r) \geq 0$), weak energy condition (WEC: $\rho(r) - p_r(r) \geq 0$ and $\rho(r) - p_t(r) \geq 0$) and the strong energy condition (SEC : $\rho(r) - p_r(r) - 2p_t(r) \geq 0$) are also satisfied by our model Fig 10. For static and stable stellar configurations the stability factor "v_t^2- v_r^2 " should lies between -1 and 0, for unstable configuration its value lies between 0 and 1. By analysing Fig. 12 we can say that our model is a stable model. At the centre of stellar object the matter shows an isotropic nature because Δ is zero there, but as *r* increases radially outward from the centre to the boundary the matter shows an increasing anisotropic behaviour. The profile of anisotropy plotted against *r* in Fig. 6 shows that anisotropic force is repulsive in nature. Everywhere within the interior of stellar structure the adiabatic index (Γ) is always greater than 4/3 and which represent a non collapsing stable model, Fig. 13. It can be observed from the graphical analysis that all the physical variables exhibit well behaved trends inside the stellar configurations for the compact star models: PSR J1614-2230, 4U1608-52 and Cen X-3. The Values of the parameters *a*, *b*, *c*, *δ*, *β*, radius (*R*), mass (*M*) for compact stars PSR J1614-2230, 4U1608-52 and Cen X-3 are given in Table 1. We conclude that all the physical properties of compact star models for PSR J1614-2230, 4U1608-52 and Cen X-3 are well behaved hence can represent the physically acceptable structures.

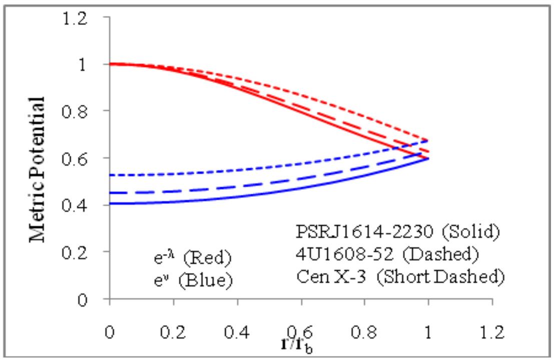

figure 1: profile of metric potentials e^{λ} and e^{ν} against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

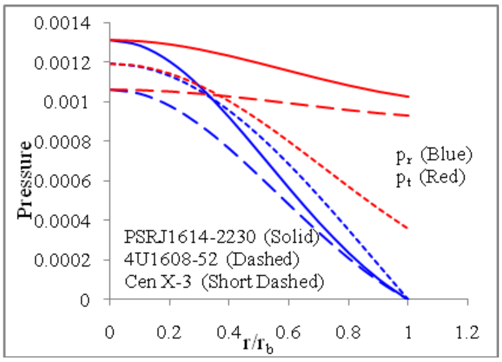

figure 2: profile of pressure against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constantsmentioned in Table 1.

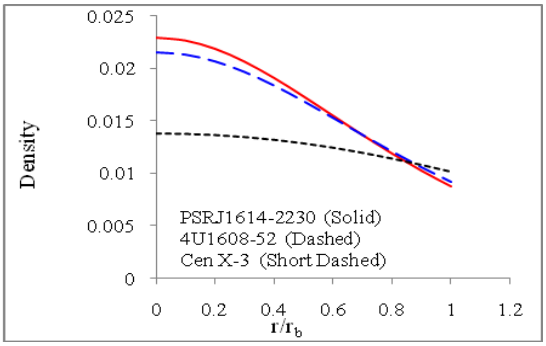

figure 3: profile of density against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

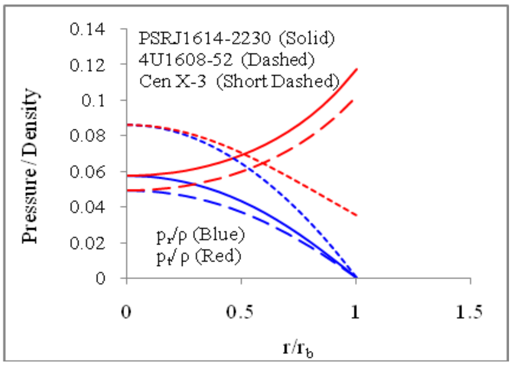

figure 4: profile of pressure to density ratio against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

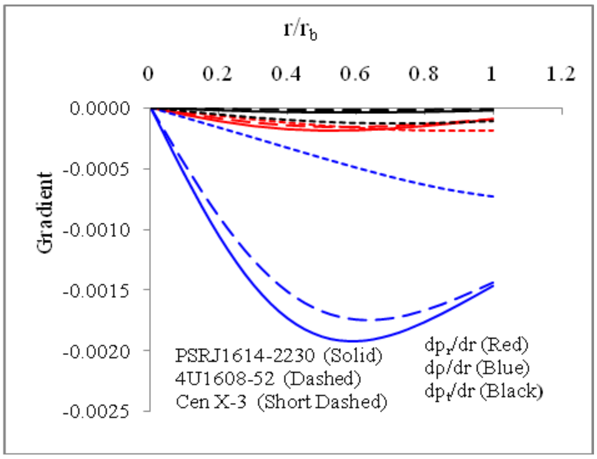

figure 5: profile of pressure and density gradients against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

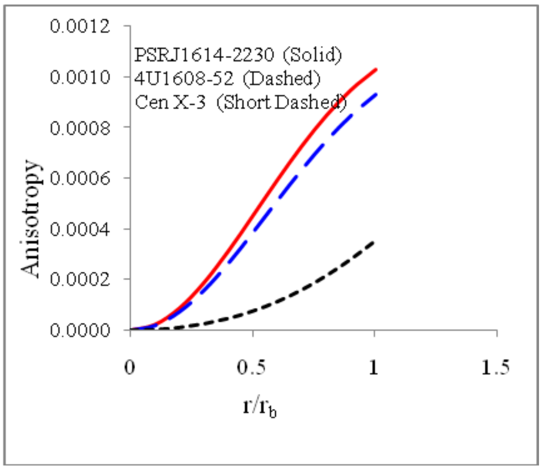

figure 6: profile of anisotropy (Δ) against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

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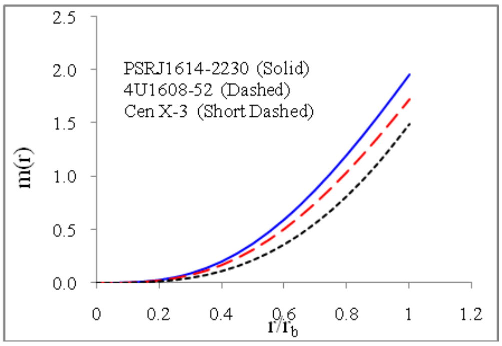

figure 7: profile of mass against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

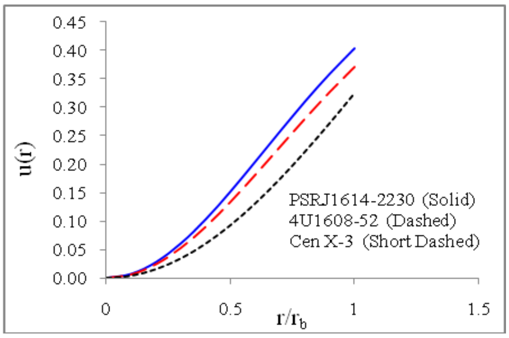

figure 8: profile of compactness parameter against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

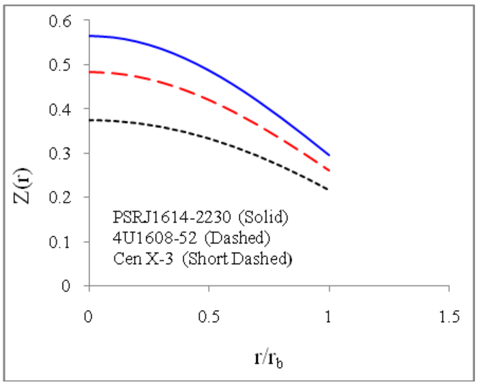

figure 9: profile of gravitational red shift against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned inTable 1.

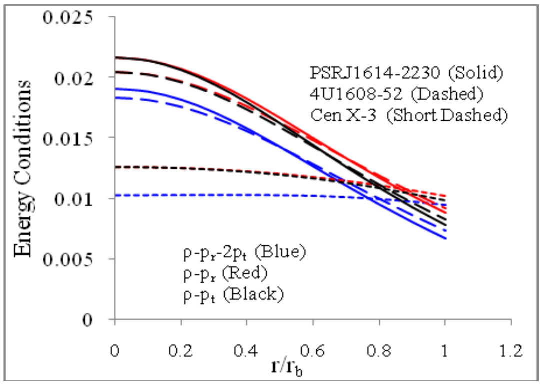

figure 10: profile of energy conditions against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

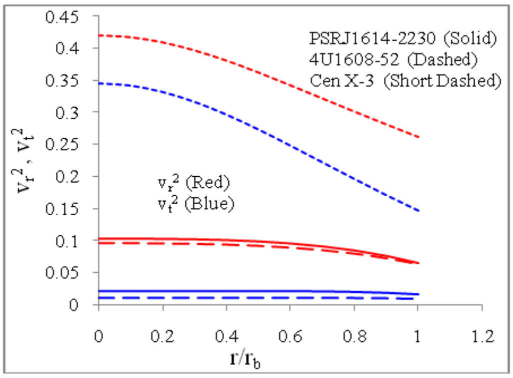

figure 11: profile of square of radial and transverse velocity of sound against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

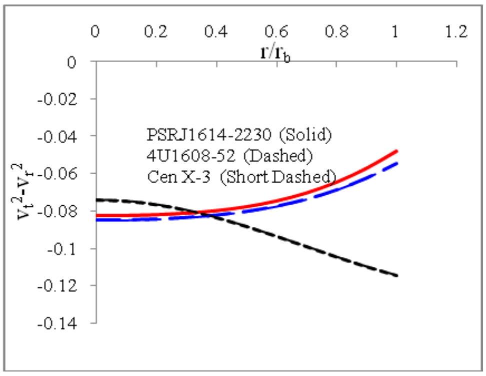

figure 12: profile of stability factor against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

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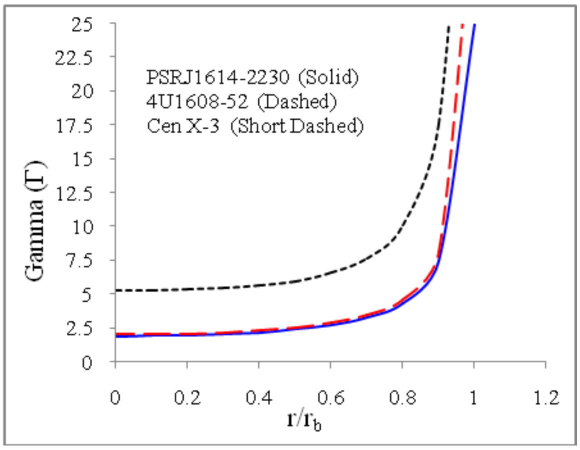

figure 13: profile of relativistic adiabatic index (Γ) against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

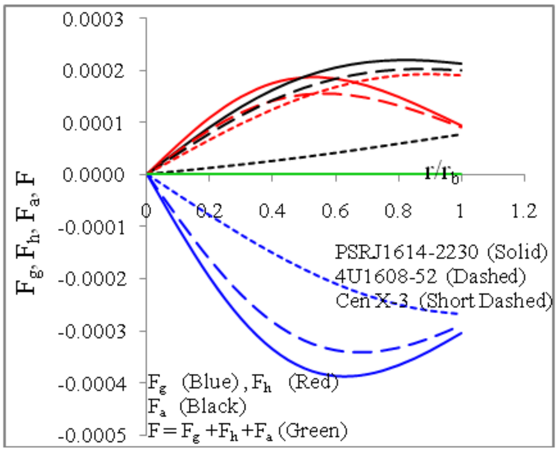

figure 14: profile of different forces against radial coordinate r/r_b for PSRJ1614-2230, 4U1608-52 and Cen X-3 for the constants mentioned in Table 1.

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