STOCHASTIC ANALYSIS OF TWO UNIT SYSTEM WITH SERVER FAILURE

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ABSTRACT

The paper analyzes the sensitivity analysis of two unit system with server failure using Regenerative Point Graphical Technique. Taking failure and repair rates constant. A state diagram of the system depicting the transition rates is drawn. Expressions for path probability mean sojourn times, mean time to system failure, availability of the system, busy period of server, expected number of servers visits are derived. Sensitive analysis of the system is done. Tables and graphs are prepared followed by conclusion.

Keyword: Busy Period Server, Expected Server unit, Mean time System Failure, Parallel Components, Stochastic analysis, Standby unit

1.INTRODUCTION

A system may consist of a number of units and individual units have great importance in a system for its proper functioning. In this paper stochastic modeling and sensitivity analysis of two units (one having subunits in parallel and other having subunits in series) system is discussed.Here for the stochastic analysis we have considered two units system A & B in which unit A having sub units in parallel hence if one/ more sub units fail then system works in reduced capacity and if the number of sub units failure is greater than a predefined number or else, then system is considered to be in the failed state. Unit B have sub units in series hence if any of its sub unit fail than the unit fail causing the whole system in failed state, but for the ease of calculations and study of effect of failure of server it is supposed further that unit B never fails. Fuzzy concept is used to decide failure/working state of units. There is single server which may also fail is discussed in analyzing system. Taking failure rates exponential (constant), repair rates general & independent and taking into consideration various probabilities, a transition state diagram of system is developed to find different level circuits and base state. Problem is attempted using RPGT to model system parameters. System behavior is discussed by drawing tables and graphs. Particular cases are taken to compare the effect of changing repair / failure rates of units while keeping failure/ repair rates respectively fixed.

2. NOTATIONS & ASSUMPTIONS

The following assumptions and notations are taken: -

α : Direct constant failure rate of unit ‘A’

α_1: Failure rate of unit ‘A’ to reduced state $\bar{A}$.

α_2: Failure rate of unit A from reduced state $\bar{A}$ to failed state ‘a’.

α_3: Failure rate of server

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β: Repair rate of unit A from failed / reduced state by the server.

β_1: Repair rate of server S.

○ Full Capacity Working State

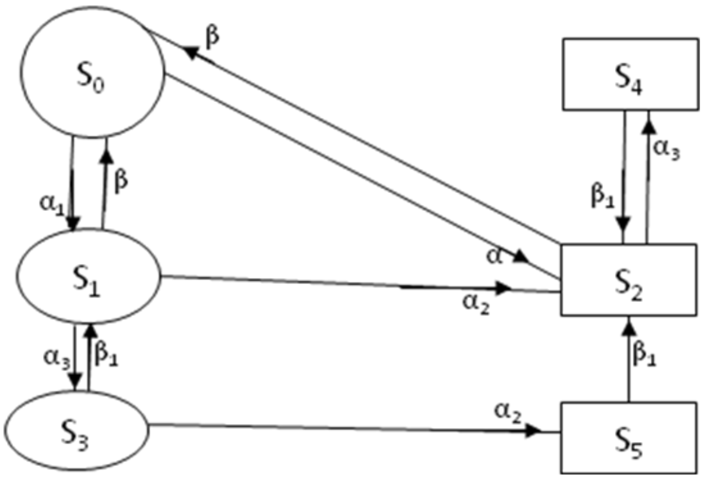
□ Failed State

A/$\bar{A}$/a : Unit in full capacity working / reduced state / failed state.

B : Unit B in full capacity working.

3. TRANSITION DIAGRAM OF SYSTEM

Consideration all assumptions & notations transition diagram of system is given in Figure 5.1



Figure

S_0 = ABS, S_1 = $\bar{A}$BS, S_2 = aBS, S_3 = $\bar{A}$Bs

S_4 = aBs, S_5 = aBs

4. Level Circuits w. r. t. the Simple Paths (Base-State '0')

Vertex j	$\left(0 \xrightarrow{S_r} j\right)$: ($P_0$)	(P_1)	(P_2)
0	$\left(0 \xrightarrow{S_1} 0\right)$: (0,1,0)	(1,3,1)	Nil
	$\left(0 \xrightarrow{S_2} 0\right)$: (0,2,0)	(2,4,2)	Nil
	$\left(0 \xrightarrow{S_3} 0\right)$: (0,1,2,0)	(1,3,1), (2,4,2)	Nil
	$\left(0 \xrightarrow{S_4} 0\right)$: (0,1,3,5,2,0)	(2,4,2)	Nil
1	$\left(0 \xrightarrow{S_1} 1\right)$: (0,1)	(1,3,1)	Nil
2	$\left(0 \xrightarrow{S_1} 2\right)$: (0,2)	(2,4,2)	Nil

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	$\left(0 \xrightarrow{S_2} 2\right)$: (0,1,2)	(1,3,1), (2,4,2)	Nil
	$\left(0 \xrightarrow{S_3} 0\right)$: (0,1,3,5,2)	(2,4,2)	Nil
3	$\left(0 \xrightarrow{S_1} 3\right)$: (0,1,3)	Nil	Nil
4	$\left(0 \xrightarrow{S_1} 4\right)$: (0,1,2,4)	Nil	Nil
	(0,2,4), (0,1,3,5,2,4)	Nil	Nil
5	$\left(0 \xrightarrow{S_1} 5\right)$: (0,1,3,5)	Nil	Nil

5. Transition Probabilities

$q_{i,j}(t)$

$q_{0,1}(t) = \alpha_1 e^{-(\alpha+\alpha_1)t}$

$q_{0,2}(t) = \alpha e^{-(\alpha+\alpha_1)t}$

$q_{1,0}(t) = \beta e^{-(\beta+\alpha_2+\alpha_3)t}$

$q_{1,2}(t) = \alpha_2 e^{-(\beta+\alpha_2+\alpha_3)t}$

$q_{1,3}(t) = \alpha_3 e^{-(\beta+\alpha_2+\alpha_3)t}$

$q_{2,0}(t) = \beta e^{-(\beta+\alpha_3)t}$

$q_{2,4}(t) = \alpha_3 e^{-(\beta+\alpha_3)t}$

$q_{3,1}(t) = \beta_1 e^{-(\beta_1+\alpha_2)t}$

$q_{3,5}(t) = \alpha_2 e^{-(\beta_1+\alpha_2)t}$

$q_{4,2}(t) = \beta_1 e^{-\beta_1 t}$

$q_{5,2}(t) = \beta_1 e^{-\beta_1 t}$

$P_{ij} = q^*_{i,j}(0)$

$p_{0,1} = \alpha_1/(\alpha+\alpha_1)$

$p_{0,2} = \alpha/(\alpha+\alpha_1)$

$p_{1,0,} = \beta/(\beta+\alpha_2+\alpha_3)$

$p_{1,2} = \alpha_2/(\beta+\alpha_2+\alpha_3)$

$p_{1,3} = \alpha_3/(\beta+\alpha_2+\alpha_3)$

$p_{2,0} = \beta/(\beta+\alpha_3)$

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$p_{2,4} = \alpha_3/(\beta+\alpha_3)$

$p_{3,1} = \beta_1/(\beta_1+\alpha_2)$

$p_{3,5} = \alpha_2/(\beta_1+\alpha_2)$

$p_{4,2} = 1$

$p_{5,2} = 1$

6. Mean Sojourn Times

$R_i(t)$

$R_0(t) = e^{-(\alpha+\alpha_1)t}$

$R_1(t) = e^{-(\beta+\alpha_2+\alpha_3)t}$

$R_2(t) = e^{-(\beta+\alpha_3)t}$

$R_3(t) = e^{-(\beta_1+\alpha_2)t}$

$R_4(t) = e^{-\beta_1 t}$

$R_5(t) = e^{-\beta_1 t}$

$\mu_i = R_i^*(0)$

$\mu_0 = 1/(\alpha+\alpha_1)$

$\mu_1 = 1/(\beta+\alpha_2+\alpha_3)$

$\mu_2 = 1/(\beta+\alpha_3)$

$\mu_3 = 1/(\beta_1+\alpha_2)$

$\mu_4 = (1/\beta_1)$

$\mu_5 = (1/\beta_1)$

8. Parameter of the system:

8.1 MTSF (T_0): States to which system can transit (initial state 0), before transiting/visiting to any failed state are j = 0, 1, 3 taking ‘ξ’ = ‘0’. Applying RPGT, MTSF is given as

$$\text{MTSF }(T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} i\right)\right\}\mu i}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} \xi\right)\right\}}{\prod_{m_2\neq\xi}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$T_0 = [(0,0)\mu+\{(0,1)\mu_1/1-(1,3,1)\}+\{(0,1,3)\mu_3/1-(1,3,1)\}]/[1-(0,1,0)$?????XXX

$/\{1-(1,3,1)\}]$

$= [\mu_0+\{p_{0,1}\mu_1/1-(1,3,1)\}+\{p_{0,1}p_{1,3}\mu_3/1-(1,3,1)\}]/[\{1-(1,3,1)-(0,1,0)\}$

$/\{1-(1,3,1)\}]$

$= [\{1-(1,3,1)\}\mu_0+p_{0,1}\mu_1+p_{0,1}p_{1,3}\mu_3]/[\{1-(1,3,1)\}-(0,1,0)]$

$= [(1-p_{1,3}p_{3,1})\mu_0+p_{0,1}\mu_1+p_{0,1}p_{1,3}\mu_3]/[1-p_{1,3}p_{3,1}-p_{0,1}p_{1,0}]$

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8.2 Availability of the System (A_0)

States at where system is available are j = 0, 1, 3 and taking ‘ξ’ = ‘0’ system availability is given by

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}fj,\mu j}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\prod_{m_2\neq\xi}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$$A_0 = \left[\sum_j V_{\xi,j}\,,f_j,\mu_j\right] \div \left[\sum_i V_{\xi,i}\,,f_j,\mu_i^1\right]$$

$$= (V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,3}\mu_3)/D$$

Where $D = (V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_1+V_{0,3}\mu_3+V_{0,4}\mu_4+V_{0,5}\mu_5)$

8.3 Busy Period of the Server

The states where server is busy are 1≤j≤5 and taking ξ = ‘0’, server remains busy applying RPGT is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\},nj}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\prod_{m_2\neq\xi}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$$B_0 = \left[\sum_j V_{\xi,j}\,,n_j\right] \div \left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

$$B_0 = (V_{0,1}\mu_1+V_{0,2}\mu_1+V_{0,3}\mu_3+V_{0,4}\mu_4+V_{0,5}\mu_5)/\ D$$

$$= 1-(V_{0,0}\mu_0/D) = 1-(\mu_0/D)$$

8.4 Expected Fractional Number of Inspections by the repair man

States where repairman visits afresh are j = 1, 2, Taking ‘ξ’ = ‘0’, Expected visits of repair man applying RPGT is as given by

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}}{\prod_{k_1\neq\xi}\{1-V_{\overline{k_1 k_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\prod_{k_2\neq\xi}\{1-V_{\overline{k_2 k_2}}\}}\right\}\right]$$

$$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

$$= (V_{0,1}+V_{0,2})/D$$

9. Particular Cases: -

Sensitivity analysis due to change in failure rates of units

Taking / fixing repair rates = 0.80 and varying the failure rates, we get following table

α_i	T_0			
	α	α_1	α_2	α_3
0.1	9.9999	9.9999	9.9999	9.9999
0.15	6.9172	9.9998	9.5111	9.9991
0.2	5.2872	9.9981	5.1715	9.9987

(T_0) Graph

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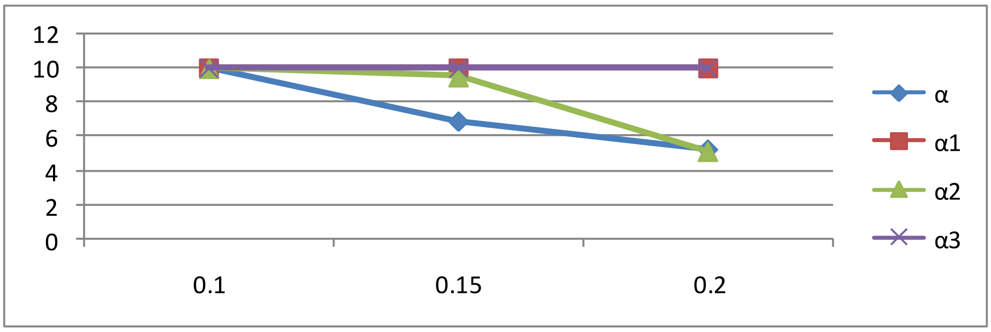


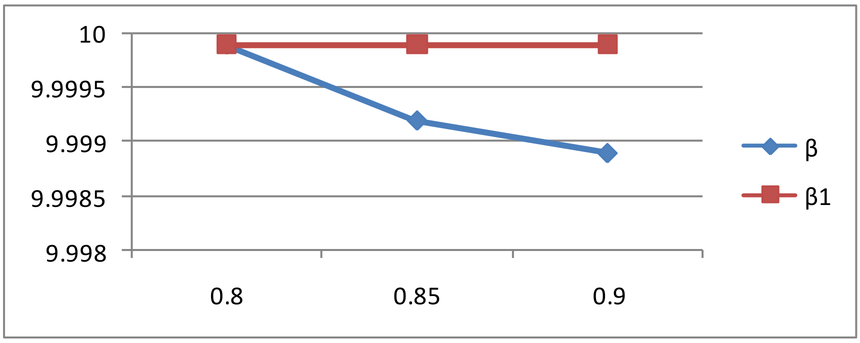
From the table and graph we see that MTSF is maximum when failure rates are minimum i.e. 0.1 while in the table looking at the columns we see that MTSF is decreases more rapidly when failure rate a i.e. major failure of unit A increases, hence to have best MTSF values management should take care / precaution that unit A does not fail directly also the server failure rate should also be minimum for optimum value of MTSF.

Sensitivity analysis due to change in repair rates of units

Taking / fixing failure rates = 0.10 and varying the repair rates, we get following table

β_i	T_0	
	β	β_1
0.80	9.9999	9.9999
0.85	9.9992	9.9999
0.90	9.9989	9.9999

(T_0) Graph



From table and graph while looking across horizontally and vertically we see that there is no increase in MTSF, hence increasing failure rates of units or server is not fruitful.

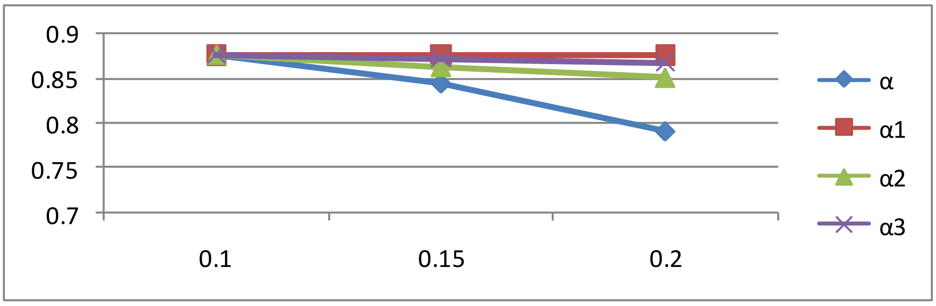
ISBN : 978-93-87793-76-7

Sensitivity analysis due to change in failure rates of units

Taking / fixing repair rates = 0.80 and varying the failure rates, we get following table

α_i	A_0			
	α	α_1	α_2	α_3
0.1	0.8771	0.8771	0.8771	0.8771
0.15	0.8447	0.8768	0.8636	0.8724
0.2	0.7912	0.8765	0.8516	0.8681

(A_0) Graph



From the table and graph we conclude that while moving in column from top to bottom availability decreases with increase in failure rates of different units but it is more effected due to direct failure of unit A, hence unit A should be arranged/ install by the management best in design and quality.

Sensitivity analysis due to change in repair rates of units

Taking / fixing failure rates = 0.10 and varying the repair rates, we get following table

β_i	A_0	
	β	β_1
0.80	0.8771	0.8771
0.85	0.8835	0.8779
0.90	0.8893	0.8785

(A_0) Graph

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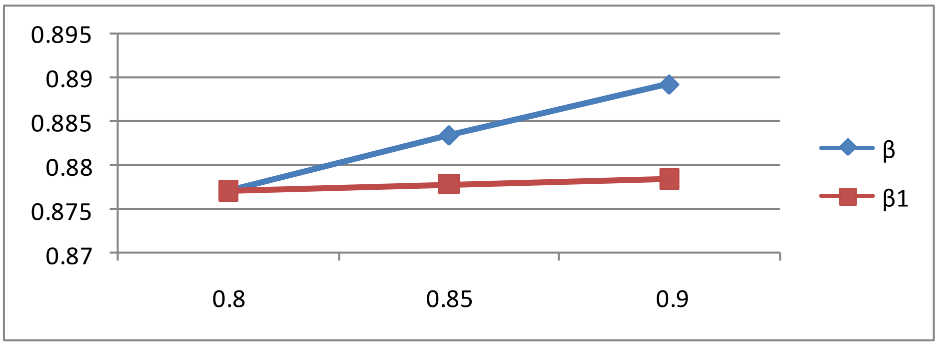


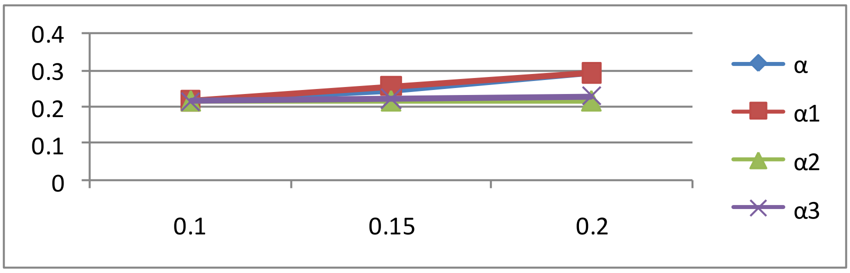
From above table we see that in moving from top downwards i.e. when the repair rates of units are being increased availability of system increase but its proportional rate of increase is more when the unit A is repaired directly.

Sensitivity analysis due to change in failure rates of units

Taking / fixing repair rates = 0.80 and varying the failure rates, we get following table

α_i	B_0			
	α	α_1	α_2	α_3
0.1	0.2173	0.2173	0.2173	0.2173
0.15	0.2463	0.2577	0.2174	0.2244
0.2	0.2940	0.2940	0.2174	0.2307

Table (B_0) Graph



From table and graph while looking from top to down as failure rate increase, busy period of server increases but proportional increase in B0 is more when the unit 'A' fails directly in comparison to it's moving to reduced state or failure of server.

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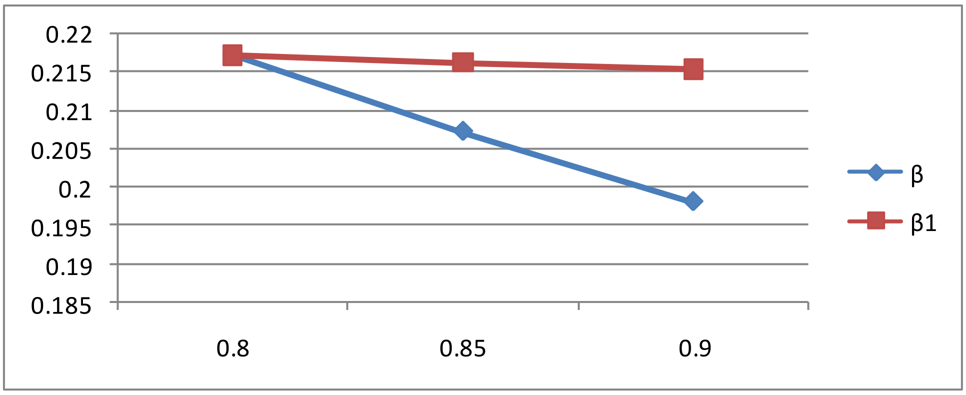
Sensitivity analysis due to change in repair rates of units

Taking / fixing failure rates = 0.10 and varying the repair rates, we get following table

β_i	B_0	
	β	β_1
0.80	0.2173	0.2173
0.85	0.2073	0.2163
0.90	0.1981	0.2154

(B_0) Graph



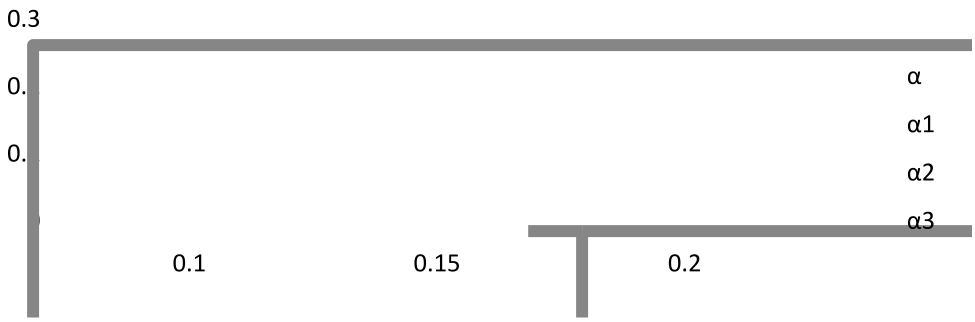
On comparing the second and third column while going top to bottom it is observed that busy period have lower values when unit A is repaired directly from the failed state to full working state.

Sensitivity analysis due to change in failure rates of units

Taking / fixing repair rates = 0.80 and varying the failure rates, we get following table

α_i	V_0			
	A	α_1	α_2	α_3
0.1	0.1845	0.1845	0.1845	0.1845
0.15	0.2063	0.2208	0.1880	0.1924
0.2	0.2458	0.2535	0.1913	0.2087

(V_0) Graph



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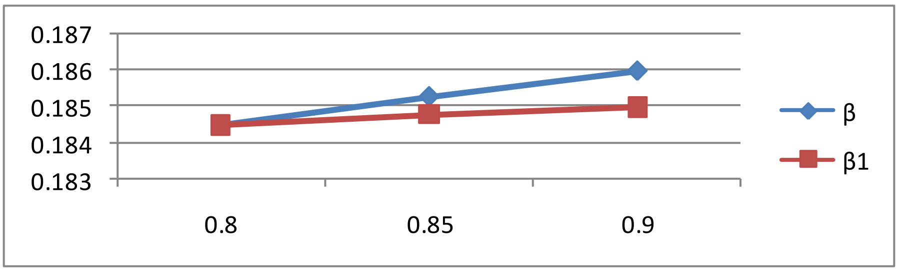
Looking the above table 5.7 and graph 5.8 fractional value of V_0 should be least for best system; the trend is shown in fourth column with increase in value of α_2 hence it is recommended to the industry personnel that failure rates of units should be least for least value of V_0.

Sensitivity analysis due to change in repair rates of units

Taking / fixing failure rates = 0.10 and varying the repair rates, we get following table

β_i	V_0	
	β	β_1
0.80	0.1845	0.1845
0.85	0.1853	0.1848
0.90	0.1860	0.1850

(V_0) Graph



From a quick look on table 5.8 and graph 5.9 there is no increase or decrease in value of V0 as whenever the system fail it is due to failure of unit or server there after server will have to be called in, hence there is no benefit to reduce V0 by employing expert repairman for units or server.

CONCLUSION:

MTSF is maximum when failure rates are minimum i.e. 0.1. We see that MTSF is decreases more rapidly when failure rate of unit A increases. Hence, to have best MTSF values management should take care that unit A does not fail directory. Also the server failure rate should be minimum for optimum value of MTSF. Availability is also decreases with increase in failure rates of different units but it is more affected due too direct failure of unit A. So, unit A should be arrange best in design and quality. When the repair rate of units are being increased, availability of system increases. When failure rate increases busy period of server increases. But proportional increases in B_0 is more when the unit A fail directly in comparison to its moving to reduced state. Also V_0 should be least for best system. It is recommended to the industry personnel that failure rates of units should be least for least value of V_0.

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