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Study of Fuzzy Domination and Chromatic Number of Fuzzy Graph

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Abstract:

Let G be a fuzzy graph without isolated vertices. The dominator coloring issue looks for an appropriate coloring of G with the extra property that each vertex in the chart rules a whole coloring class. In this paper, another idea of fuzzy dominator coloring and fuzzy chromatic number is presented on Cartesian result of straightforward fuzzy diagram, for example, fuzzy way, fuzzy cycle and complete fuzzy chart. Fuzzy dominator coloring, chromatic number of fuzzy complete diagram, fuzzy way, and fuzzy cycle has been talked about. Limits for fuzzy dominator coloring, chromatic number have been found. The connection between control number, fuzzy dominator coloring and chromatic number is determined.

Keywords: Fuzzy graph, domination number, chromatic number, fuzzy dominator coloring.

1. Introduction:

Graph is a simple model of relation and it is a convenient way of representing information involving relationship between objects. The article is spoken to by vertices and relations by edges. When there is dubiousness in the portrayal of the articles or in its relationship or in both we have to plan fuzzy diagram model. One of the most significant properties of fuzzy diagram model is fuzzy chart coloring which is utilized to take care of issues of combinatorial enhancement like traffic signal control, test booking, register designation and so on In these applications, fuzzy dominator coloring assumes an essential job. The idea of fuzzy Dominator coloring was presented by Ralucca Michelle Gera in 2006 [1]. In a similar time, the fuzzy coloring of a fuzzy diagram was characterized by the creators in Eslachi and Onagh[2].

At that point pourspasha [3] likewise acquainted various methodologies with coloring the fuzzy chart. In fuzzy chart, it is essential to recognize the idea of circular segments. In this paper, we think about all circular segments as solid curves for determining the limits in hypothesis. In this paper we decide the fuzzy dominator coloring, fuzzy chromatic number for cartesian result of fuzzy way, fuzzy cycle and complete fuzzy chart. The fuzzy dominator coloring of a diagram G×H is an appropriate coloring where every vertex of the chart overwhelms each vertex of atleast one coloring class. The base number of tones required for a fuzzy dominator coloring of G×H is known as the fuzzy dominator chromatic number and it is meant by fd (G×H).

2. Related Definitions

In this section, basic concepts of fuzzy graph and coloring are discussed. Notation and more formal definitions which are followed as in [1], [2], [4], [10].

1) A fuzzy graph $G=(\sigma, \mu)$ is a pair of functions $\sigma: V\rightarrow [0, 1]$ and $\mu:V\times V\rightarrow[0, 1]$, where for all $u, v\in V$, we have $\mu(u, v)\leq \sigma(u)\wedge\sigma(v)$.

2) The order p and size q of a fuzzy graph $G=(\sigma, \mu)$ are defined to be $p = \sum_{x\in V} \sigma(x)$ and $q = \sum_{xy\in V} \mu(xy)$.

3) Two vertices u and v in $\hat{G}$ are called adjacent if $(½)[\sigma(u) \wedge \sigma(v)] \leq \mu(uv)$.

4) An arc (u, v) is said to be a strong arc or strong edge, if $\mu(u, v) \geq \mu^{\infty}(u, v)$ and the node v is said to be the strong neighbor of u. A node u is said to be isolated if $\mu(u, v) = 0$ for all $u \neq v$. In a fuzzy graph, every arc is a strong arc then the graph is called thestrong arc fuzzy graph.

5) u is a node in fuzzy graph G then N(u) = {v: (u, v) is a strong arc} is called the neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called closed neighborhood of u.

6) A path in which every arc is a strong arc then the path is called strong path and the path contains n strong arcs is denoted by P_n.

7) A cycle in G is said to be fuzzy cycle if it contains more than one weakest arc.

8) A fuzzy graph $G=(\sigma, \mu)$ is called complete fuzzy graph if $\mu(u, v)=\sigma(u)\wedge\sigma(v)$, for all $u, v\in V$ and is denoted by K_σ.

9) Two nodes of a fuzzy graph are said to be fuzzy independent if there is no strong arc between them. A subset S of V is said to be fuzzy independent of if any two nodes of S are fuzzy independent.

10) G is a fuzzy graph on V and $S \subseteq V$, then the fuzzy cardinality of S is defined to be $\Sigma_{v\in S}\, \sigma(v)$.

11) A dominating set D of fuzzy cardinality $|D|= \Sigma\ \sigma(u)$, for all $u\in D= \gamma(G)$ is called minimum dominating set or γ-set. A dominating set D of a fuzzy graph G is called the minimal dominating set if and only f for each vertex $v\in V$, D-{u} is not a dominating set of G. The minimum fuzzy cardinality taken over all minimal

dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$.

12) A fuzzy dominator coloring (FDC) of a fuzzy graph G is a proper fuzzy coloring in which each vertex of G dominates every vertex of atleast some color class.

13) Fuzzy dominator chromatic number of a fuzzy graph is the minimum number of color classes in a dominator fuzzy coloring of G. It is denoted by χ_{fd} (G).

3. Domination and Chromatic number of Cartesian product of fuzzy graphs

In this paper, the Cartesian product on same type of two fuzzy graphs (say G and H) such as fuzzy path, fuzzy cycle and complete fuzzy graph [5]. The order of H should be greater than or equal to the order of G. Let $G=(V_1, \sigma_1, \mu_1)$ be a fuzzy graph and $H==(V_2, \sigma_2, \mu_2)$ be a fuzzy graph. The Cartesian product $G\times H=(V, X)$ of fuzzy graph G and H. where $V=V_1\times V_2$, $X=\{((u, u_2), (u, v_2))| u\in V_1, (u_2, v_2)\in X_2\}\cup\{((u_1, w), (v_1, w)) | w \in V_2, (u_1, v_1)\in X_1\}$. Using these notations [6,7], the following theorems are defined. To derive the theorem we use independent fuzzy path and fuzzy cycles which does not have unique strong neighbors in fuzzy path and fuzzy cycle of G×H.

In 1941, Brook's gave an upper bound for the chromatic number in case of crisp graphs and stated that 'For a connected graph which is neither an odd cycle nor a complete graph, the chromatic number $\chi(G) \leq \otimes(G)$'[8].

Proposition:

If G and H are fuzzy path on m and n vertices and the cartesian product G×H of G and H is a fuzzy graph on mn vertices [9]. Then $\chi_f(G\times H)=2$.

Proof:

In G×H, consider a fuzzy path with distinct vertices say $P_1= \{(u_1, v_1), (u_1, v_2),.. (u_1, v_n)\}$, $P_2= \{(u_2, v_1), (u_2, v_2),.., (u_2, v_n)\}$, ….., $P_m= \{(u_m, v_1), (u_m, v_2), …, (u_m, v_n)\}$. The vertices in P_1 assign color1 and color2 alternatively. Depending upon the color of verticesin P_1, assign alternate color class 2 and color1 in P_2 and so on.

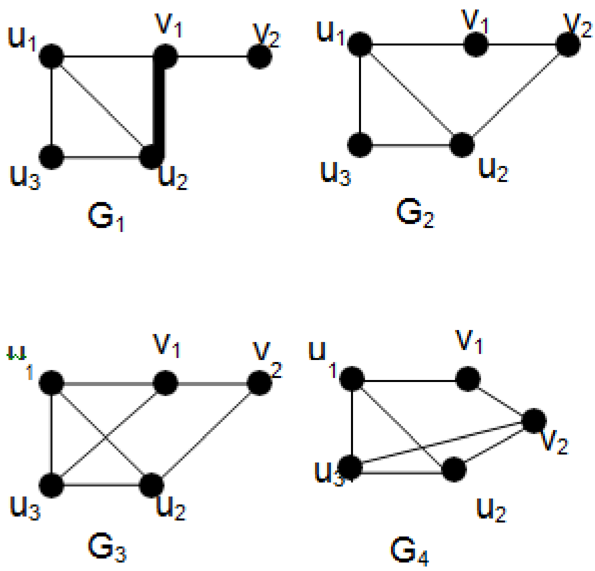
The two independent fuzzy paths are $P_1= \{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4)\}$, $P_2= \{u_2, v_1), (u_2, v_2), (u_2, v_3), (u_2, v_4)\}$. Color 1 and color 2 are assigned to the vertices of P_1alternatively. However, color 2 and color 1 are assigned to the vertices in P_2alternatively. Thus $\chi_f(G\times H)=2$ [11].

Consider the fuzzy graph of G×H. The fuzzy chromatic number χ_f (G×H)=2 since there exists a family Γ={< $_1$, < $_2$}. Satisfying the definition of fuzzy coloring for fuzzy graph G×H is shown in the table below [12].

Table 1: Fuzzy coloring of $P_2 \times P_4$

Vertices	$_1$	$_2$	**Maximum**
(u_1, v_1)	0. 5	0	0. 5
(u_1, v_2)	0	0. 3	0. 3
(u_1, v_3)	0. 6	0	0. 6
(u_1, v_4)	0	0. 4	0. 4
(u_2, v_1)	0	0. 5	0. 5
(u_2, v_2)	0. 5	0	0. 5
(u_2, v_3)	0	0. 5	0. 5
(u_2, v_4)	0. 4	0	0. 4

Theorem 2.6 For any connected fuzzy graph G, (G)+(G) = 2n-5 for any n>4, if and only if G is isomorphic to K_6, $K_3(P_3)$, $K_3(1,1,0)$, P_5, $K_4(1,0,0,0)$, $K_{1,3}$ (or) any one of the following fuzzy graphs in the figure below [13].



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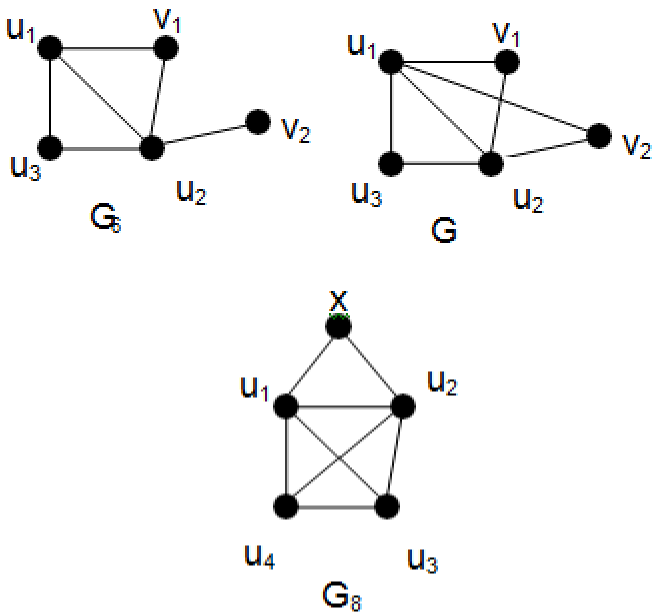

Figure 1 [14]

Proof: If G is any one of the graphs in the theorem, then it can be verified that (G)+ (G)=2n-5.
Conversely set
(G)+ (G)=2n-5 then, (G)=n and (G)=n-5 (or) (G)=n-1 and (G)=n-4 (or) (G)=n-2 and (G)=n-3 (or) (G)=n-3
and (G)=n-2 (or) (G)=n-4 and (G)=n-1 (or) (G)=n-5 and (G)=n

CONCLUSION

In this paper, upper bound of the sum of fuzzy independent domination and chromatic number is studied. The outcomes can be stretched out to different mastery boundaries. The structure of the charts had been given in this paper can be utilized in models and organizations. The creators have acquired comparative outcomes with enormous instances of fuzzy diagrams.

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