Some numerical methods to solve partial differential equations

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ABSTRACT:

Numerical methods often are of a repetitive nature. These consists in repeated execution of the same process where at each step the result of preceding step is used .this process is known as iteration process and is repeated till the result is obtained to a desired degree of accuracy.To improve the quality of convergence and accuracy of the given results implementation of numerical methods becomes the essential tool to handle with. Here we are providing the basic methods of numerical analysis.

Solution of Algebraic and Transcendental Equations:-

Generally we are interested in finding the roots of complicated equation occurring in engineering and sciences of the type

F(x)=0

we are here discussing some such methods:-

Bisection Method:-Generally root of the equation F(x)=0 lies between such kinds of a and b in which function is continuous and f(a) and f(b) are of opposite signs. Then we can say that a root will lies between a and b such that first approximation to the root is $c=\frac{a+b}{2}$.if f(c)=0 then c is the root of f(x),unless the root will lies between a and c or b and c according as f(c) is positive or negative.

This method is little slow to have and rate of convergence is main limitation of this method.

Regula Falsi Method:-this method is also known as method of false position and is having some similarity with the bisection method due to its classical nature. In this method we choose a and b such that f(a) and f(b) are of opposite signs .

i.e. f(a) .f(b)<0

for the continuation of iteration we are using the symbol x_0 and x_1 instead of a and b

so for the iteration process method is

$x_2 = x_0 - \frac{x_0 - x_1}{f(x_0) - f(x_1)} f(x_0)$

Which is approximation to the root. If $f(x_0)$ and $f(x_2)$ are of opposite signs,the the root will lies between (x_0) and x_2.so replacing (x_1) and x_2 in the iterative method we obtain the approximation x_3.we will apply the method until the repetition of roots does not exists.

However convergence of this method is little faster than that of bisection method. But this convergence is not the better one to have.

Newton –Raphson method :-This method is much faster method as compared to bisection and regula falsi method. In the starting of the method we have to find two no.s a and b such that f(a) and f(b) are of opposite signs. Then we have to select that value for x_0 which is closer to zero . thus a closer approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f\prime(x_0)}$$

Similarly for the second iteration Newton raphson formula becomes

$x_2 = x_0 - \frac{f(x_1)}{f\prime(x_1)}$ continuing like that the general formula becomes

$$x_n = x_0 - \frac{f(x_n)}{f\prime(x_n)}.$$

the process is repeated till the value of x did not converge to the original value.

some of the iterative formulas derived from Newton raphson method:-

1 iterative formula for 1/N is $x_{n+1} = x_n(2 - N x_n)$

2. iterative formula for $\sqrt{N}$ is $x_{n+1} = \frac{1}{2}(x_n - N/x_n)$

3. iterative formula for1/ $\sqrt{N}$ is $x_{n+1} = \frac{1}{2}(x_n + 1/N x_n)$

Similarly we can derive so many iterative formulas from the Newton raphson formula .doe to these deductions convergence becomes very fast and approximation to root is good.

Solution of linear simultaneous equations:-to solve the linear simultaneous equation a lots of complicated calculation have to perform ,to avoid these difficulties in numerical analysis some methods have been discovered .some of these methods are discussed here:-

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Direct methods of solution:-

1Gauss elimination method:-this method is successful in three or four variables and equations but in higher system this method becomes tedious to solve. this method mainly makes use of elementary matrix operation or can be solved by elilminating the variables successively and system is reduced to the upper triangular matrix .after finding the value of one variable by making use of back substitution we can find value of other variables

Consider a system of equations

a_1 x+b_1 y+c_1 z=d_1

a_2 x+b_2 y+c_2 z=d_2

a_3 x+b_3 y+c_3 z=d_3

First of all we eliminate coefficient of x then eliminate the coefficient of y and consequently we have remained coefficient of z and we can easily find out the value of z. At last form of equation is:

a_1 x+b_1 y+c_1 z=d_1

b'_2 y+c'_2 z=$d_2{}'$

c'_3 z=$d_3{}'$

Thus after finding value of z we can easily find out value of x and y by back substitution.

Gauss –Jordan method-this method was not a different method but was a simple modification in the concept of gauss elimination method. In this method elimination of unknown is performed not only in the equation below but in the equation above also. At last equation is reducible to the diagonal form

Iterative method of solution of solution:-

these methods provides better approximations and solutions as compared to preceding methods of system of linear equation, in the iterative methods we start the approximation from the true solution and obtain the better and better approximations from a computational cycle as many times so as to get desired accuracy

1.Jacobi iteration method:- consider the equation

a_1x+b_1y+c_1z=d_1

a_2x+b_2y+c_2z=d_2

a_3x+b_3y+c_3z=d_3

Then the next iteration can written as:-

x=k_1-l_1y-m_1z

y=k_2-l_2x-m_2z

z=k_3-l_3x-m_3y

continuing like that we have to apply iteration until value f roots or solution get very closer to real value.

Gauss seidel iteration method: This method is the modified version of the Jacobi iteration method. in the Jacobi method all the values are first calculated and then put all the values simultaneously in the second iteration whereas in this method latest values of x ,y and z are then substituted in the same iteration of next step thus we may write

x_1=k_1,

y_1=k_2-l_2 x_1-m_2z_0

z_1=k_3-l_3 x_1-m_3y_1

This method is relatively faster than the Jacobi iteration method

Relaxation method:-consider the system of equation

a_1x+b_1y+c_1z=d_1

a_2x+b_2y+c_2z=d_2

a_3x+b_3y+c_3z=d_3

We define residuals R_x, R_y, R_z by the relations

R_x=$d_1 - a_1$x+b_1y+c_1z

R_y=$d_2 - a_2$x+b_2y+c_2z

R_z=$d_3 - a_3$x+b_3y+c_3z

Numerical integration :-

the process of evaluating a definite integral from a set of tabulated values of the integrand f(x) is called numerical integration. This process when applied to a function of a single variable , is known as quadrature

Newton cotes quadrature formula:-

let $I = \int_a^b f(x)dx$

where f(x) takes the values $y_{0,}y_{1,}y_2 \dots\dots\dots\dots..y_n$ for $x = x_{0,}x_{1,}x_2 \dots\dots\dots\dots..x_n$.

let us divide the interval (a,b) into sub-intervals of width h so that $x_{0,} = a, x_{1,} = x_{0,} + h$ $_2 \dots\dots\dots\dots..x_{n=x_{0,}+nh}$ then following methods can be deducted from the Newton cotes formula

1.**Trapezoidal method:-**by substituting the n=1 in the Newton cotes formula we hve the formula

$\int_a^b f(x)dx$=$\frac{h}{2}${($y_0 + y_n$)+2($y_{1,} + y_2 + \cdots \dots\dots\dots.y_{n-1}$)}

2.**Simpson's one third method:-**by substituting n=2 in the Newton's cotes formula we can find out the required one third formula

$\int_a^b f(x)dx$=$\frac{h}{3}${($y_0 + y_n$)+4($y_{1,} + y_3 + \cdots \dots\dots\dots.y_{n-1}$) +2($y_{2,} + y_4 + \cdots \dots\dots\dots.y_{n-2}$)}

3. **Simpson's three eight method:-**by substituting n=3 in the Newton's cotes formula we can find out the required three eight formula.

$\int_a^b f(x)dx$=3 $\frac{h}{8}${($y_0 + y_n$)+3($y_{1,} + y_2 + \cdots \ldots \ldots \ldots . y_{n-1}$) +2($y_{3,} + y_6 + \cdots \ldots \ldots \ldots . y_{n-3}$)}

4.**Weddle's rule:-** by substituting n=6 in the Newton's cotes formula we can find out the required three eight formula.

$\int_a^b f(x)dx$=3 $\frac{h}{10}${($y_0 + 5y_1$+$y_{2,} + 6y_3 + \cdots$}

Conclusion:-Here we discussed the appropriate class of the numerical methods which are frequently used in the accurate solution of the problem. But still knowledge of correct implementation is necessary and which method is better in given conditions is also an issue. As we have discussed the cases of Newton's cotes method to solve numerical integration different methods i.e. trapezoidal methods, Simpson's one third and three eight methods ,Weddle methods give slight difference in the results of same problem because of their different approach of values assigned in the Newton's cotes formula.

A quick glance of solution of equation by gauss elimination, gauss Jordan in sequence with iteration methods like Jacobi and modified with seidel method gives the perfect approach to the numerical methods. Although some modifications are always expected due to some errors remained either in the form of observations or in calculating approach of numerical methods. so future of numerical methods require more speedy calculations and fast convergence as like as bisection to newtons method.

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