An introduction to differential equations

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ABSTRACT:-

Whole of the physics and engineering depends upon differential equations , their laws dominate electronics ,mechanical and civil engineering .independent use of differential equations makes it perfect tool for the use of applied mathematics. Here we are discussing some aspects of the differential equations their definitions ,types of solutions ,their applications and uses.

Differential Equation:-An equation involving independent ,dependent variable and derivative of dependent variable with respect to independent variables is called differential equation.

It can be further divided into two categories:

1.O.D.E.

2. P.D.E

1.O.D.E.(ORDINARY DIFFERENTIAL EQUATION)-A differential equation involving derivative with respect to a single independent variable is called an ordinary differential equation. for example

$\frac{dy}{dx}$+4y=sinx and$\frac{dy}{dx}$+2xcosxy=xe^x are ordinary differential equations.

Ordinary differential equations are further divided into many branches as follows-

(a) exact differential equations

(b) orthogonal trajectories

(c) linear homogeneous differential equation with constant coefficients.

(d) Ordinary simultaneous equations

2.P.D.E.(PARTIAL DIFFERENTIAL EQUATION)-A differential equation involving partial derivatives with respect to more than one independent variables is called a partial differential equation.

for example$\left(\frac{\partial y}{\partial x}\right)$+$\left(\frac{\partial y}{\partial t}\right)$+5cosx=0 is partial differential equation. Here we are providing some basic definitions of ingredients used in differential equations:-

ORDER OF A DIFFERENTIAL EQUATION :-The order of highest order derivative involved in a differential equation is called the order of the differential equation. for example order of dy=(x+sinx) dx is 1. and order of $\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+\left(\frac{dy}{dx}\right)^5=e^t$ is 3

DEGREE OF A DIFFERENTIAL EQUATION:-The degree of a differential equation is the degree of highest derivative which occurs in it,after the differential has been made free from radicals and fractions as far as the derivatives are concerned.for example degree of $\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+\left(\frac{dy}{dx}\right)^5=e^t$ is 1 as it is the degree of highest order derivative occurring in the equation.

LINEAR AND NON LINEAR DIFFERENTIAL EQUATIONS:-A differential equation is said to be linear if :-

(I) Every dependent variable and every derivative involved occur in the first derivative only.

(II) No products of dependent variables and/or derivative occur.

For example dy=(x+sinx)dx is the linear differential equation.

A differential equation which is not linear is called nonlinear differential equation. For example $\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+\left(\frac{dy}{dx}\right)^5=e^t$ is non linear differential equation.

SOLUTION OF A DIFFERENTIAL EQUATION:-Any relation between the dependent variables and independent variables, when substituted in the differential equation, reduces it to an identity is called a solution or integral of the differential equation .it should be noted that a solution of a differential equation does not involves the derivatives of the dependent variable.

For example y=ce^{2x} is the solution of $\frac{dy}{dx}$=2x because by substituting y=ce^{2x} and $\frac{dy}{dx}$=2ce^{2x} the given differential equation reduces to the identity 2ce^{2x}=2ce^{2x}.

PARTICULAR SOLUTION AND SINGULAR SOLUTION:-

Let F(x,y, $y_1, y_2 \dots, y_n$)=0… (1)

be the nth order ordinary differential equation.

- **GENERAL SOLUTION:-**
  A solution of (1) containing n independents arbitrary constants is called a general solution of the differential equation.
- **SINGULAR SOLUTION:-**A solution of (1) obtained from a general solution of (1) by giving particular values to one or more of the n independent arbitrary constants is called the singular solution of (1).

- A delay differential equation (DDE) is a type of equation which contains function of a single variable, usually called **time**, in which the derivative of the function at a certain time is given in terms of the values of the function at earlier times.
- A stochastic differential equation (SDE) is a type of equation in which the unknown quantity is a stochastic process and the equation involves some known stochastic processes, for example, the Wiener process in the case of diffusion equations.
- A differential algebraic equation (DAE) is a differential equation comprising differential and algebraic terms, given in implicit form.

Applications

Physics

- Euler–Lagrange equation in classical mechanics
- Hamilton's equations in classical mechanics
- Radioactive decay in nuclear physics
- Newton's law of cooling in thermodynamics
- The wave equation
- The heat equation in thermodynamics
- Laplace's equation, which defines harmonic functions
- Poisson's equation
- The geodesic equation
- The Nervier–Stokes equations in fluid dynamics
- The Diffusion equation in stochastic processes
- The Convection–diffusion equation in fluid dynamics
- The Cauchy–Riemann equations in complex analysis
- The Poisson–Boltzmann equation in molecular dynamics
- The shallow water equations
- Universal differential equation
- The Lorenz equations whose solutions exhibit chaotic flow.
- Ground water equations
- Current voltage relationship
- Signal systems
- Maxwell’s equations

Classical mechanics

All we know that, Newton's second law is sufficient to describe the motion of a particle. Once independent relations for each force acting on a particle are available, they can be substituted into Newton's second law to obtain an ordinary differential equation, and various other relations which are called the *equation of motion*.

Biology

Predator-prey equations

The Lotka–Volterra equations, which are also known as the predator–prey equations, are a pair of first-order, non-linear, differential equations frequently used to describe the population dynamics of two species that interact, one as a predator and the other as prey.

Chemistry

The *rate law* or rate equation for a chemical reaction is a differential equation that links the reaction rate with concentrations or pressures of reactants and constant parameters (normally rate coefficients and partial reaction orders). To determine the rate equation for a particular system one combines the reaction rate with a mass balance for the system. The concept of rate of reaction can be applied to the tumor clots in which we can minimize the rate of growth of tumor with respect to time and we can have more time to cure that by proper medicine

Conclusion:-

The study of differential equations is a wide field because of their adaptability in pure and applied mathematics, physics, and engineering. All of these branches are concerned with the properties of differential equations of various types.. Differential equations play a vital role in modeling virtually every physical, technical, or biological process,. Differential equations such as those used to solve real-life problems may not necessarily be directly solvable, i.e. do not have closed form solutions. Instead, solutions can be approximated using numerical methods and grid generation methods various fundamental laws of physics and chemistry can be obtained from differential equations. In biology and economics, differential equations are used to model the behavior of complex systems. Theory of differential equations in mathematical form is first developed together with the sciences where the equations had originated and where the results found application. However, diverse problems, sometimes originating in quite separate scientific fields, which give rise to identical differential equations. Whenever this happens, mathematical theory behind the equations can be viewed as a unifying principle behind diverse phenomena.

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