Some g-Binary θ-Closed Sets and Maps

Nazir Ahmad Ahengar[1], J.K.Maitra[2]

[1,2] Department of Mathematics and Computer Sciences R.D.V.V Jabalpur (M.P) India

Abstract— *Recently the authors [6] have introduced the concept of g-binary continuity in g-binary topological spaces. In this paper we introduce and study the new concept of θ-closed sets and g-binary θ-continuous mapss in g-binary topological spaces. We further examine various relationships between these maps and their relationship with some other maps.*

***Keywords*— g-binary closed, g-binary α-closed, g-binary semi-closed, g-binary pre-closed, g-binary θ-closed, g-binary θ-α-closed, g-binary θ-semi-closed, g-binary θ-pre-closed, g-binary θ-r-closed, g-binary θ-continuous, g-binary θ-α-continuous, g-binary θ-semi-continuous, g-binary θ-pre-continuous, g-binary θ-r-continuous**

1. INTRODUCTION

In 2011 S. Nithyanantha Jothi and P.Thangavelu [19] introduced the concept of binary topology and discussed some of its basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Nazir Ahmad Ahengar and J.K.Maitra [6] introduced the concept of g-binary continuity and studied various types of continuities in g-binary topological spaces. Further the authors Nazir Ahmad Ahengar and J.K. Maitra [9] introduced the concept of g-Binary θ-semi-continuous functions in g-binary topological spaces. The purpose is to introduce and study the new concept of θ-closed sets and g-binary θ-continuous maps in g-binary topological spaces and investigate the relationships between these maps.

Let X and Y are any two non-empty sets. A generalized binary topology (or g-binary topology) from X to Y is a binary structure $M_g \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:

i) $(\emptyset, \emptyset)$ and $(X, X) \in M_g$

ii) If $\{(A_\alpha, B_\alpha); \alpha \in \Delta\}$ is a family of members of M_g, then $(\cup_{\alpha\in\Delta} A_\alpha, \cup_{\alpha\in\Delta} B_\alpha) \in M_g$

If M_g is a generalized binary topology from X to Y, then the triplet (X, Y, M_g) is called a generalized binary topological space (g-binary topological space) and the members of M_g are called the g-binary open subsets of the g-binary topological space (X, Y, M_g). The elements of $X \times Y$ are called the g-binary points of g-binary topological space (X, Y, M_g). Let X and Y be any two non-empty sets and (A, B), (C, D) belongs to $\wp(X) \times \wp(Y)$, we say (A, B) ⊆ (C, D) if A ⊆ C and B ⊆ D. Let (X, Y, M_g) be a g-binary topological space and $A \subseteq X, B \subseteq Y$. Then (A, B) is g-binary closed in (X, Y, M_g) if $(X\backslash A, Y\backslash B) \in M_g$.

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Section 2 deals with the basic concepts of g-binary topology. In section 3 the concept of θ-closed sets are introduced and studied some properties. In section 4 the concept of g-binary θ-ccontinuous maps in g-binary topological spaces are introduced and various relationships are also established. Throughout the paper $\wp(X)$ denotes the power set of X.

2. PRELININARIES

Definition 2.1: A subset (A, B) of a g-binary topological space (X, Y, M_g) is called

i) g-binary semi-closed if $gbint(gbcl(A, B)) \subseteq (A, B)$

ii) g-binary pre-closed if $gbcl(gbint(A, B)) \subseteq (A, B)$.

iii) g-binary α-closed if $gbcl(gbint(gbcl(A, B))) \subseteq (A, B)$

iv) g-binary β-closed if $gbint(gbcl(gbint(A, B))) \subseteq (A, B)$.

v) g-binary regular-closed if $(A, B) = gbcl(gbint(A, B))$.

Definition 2.2: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. Let $f: Z \to X \times Y$ be a map. Then f is called g-binary continuous if $f^{-1}(A, B)$ is g-open (g-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 2.3: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the mapping $f: Z \to X \times Y$ is said to be g-binary semi-continuous if $f^{-1}(A, B)$ is g-semi-open (g-semi-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 2.4: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the mapping $f: Z \to X \times Y$ is said to be g-binary pre-continuous if $f^{-1}(A, B)$ is g-pre-open (g-pre-closed) in (Z, τ) for every g-binary open (g-binary open) set (A, B) in (X, Y, M_g).

Definition 2.5: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the mapping $f: Z \to X \times Y$ is said to be g-binary α-continuous if $f^{-1}(A, B)$ is g-α-open (g-α-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 2.6: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the mapping $f: Z \to X \times Y$ is said to be g-binary β-continuous if $f^{-1}(A, B)$ is g-β-open (g-β-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

3. g-BINARY θ-CLOSED SETS

Definition 3.1: Let (X, Y, M_g) be an g-binary topological space and (A, B) be a subset of $\wp(X) \times \wp(Y)$, then $gbcl_\theta(A, B) = \{(x, y) \in \wp(X) \times \wp(Y) : gbcl(U, V) \cap (A, B) \neq \emptyset, (U, V) \in M_g \text{ and } (x, y) \in (U, V)\}$. A subset (A, B) of a (X, Y, M_g) is called g-binary θ-closed if $(A, B) = gbcl_\theta(A, B)$.

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Definition 3.2: A subset (A, B) of a g-binary topological space (X, Y, M_g) is called

i) g-binary θ-semi-closed if $gbint(gbcl_\theta(A, B)) \subseteq (A, B)$.

ii) g-binary θ-pre-closed if $gbcl(gbint_\theta(A, B)) \subseteq (A, B)$.

iii) g-binary θ-α-closed if $gbcl(gbint(gbcl_\theta(A, B))) \subseteq (A, B)$.

iv) g-binary θ-β-closed if $gbint(gbcl(gbint_\theta(A, B))) \subseteq (A, B)$.

Remark 3.1: Every g-binary closed set is g-binary θ-closed but not converse as shown in Example 3.1.

Example 3.1: Let $X = \{1,2,3\}$ and $Y = \{a, b, c\}$. Then $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a, b\}), (\{2,3\}, \{c\}), (\{2\}, \{Y\}), (X, Y)\}$. Clearly M_g is g-binary topology from X to Y. Therefore the set $(\{1,2\}, \{a, b\})$ is g-binary θ-closed but not g-binary closed.

Remark 3.2: Every g-binary closed set is g-binary θ-semi-closed but not converse as shown in Example 3.2

Example 3.2: In Example 3.1 the set $(\{1,2\}, \{a, b\})$ is g-binary θ-semi-closed but not g-binary closed.

Remark 3.3: Every g-binary closed set is g-binary θ-α-closed but not converse as shown in Example 3.3.

Example 3.3: In Example 3.1 the set $(\{1,2\}, \{a, b\})$ is g-binary θ-α-closed but not g-binary closed.

Remark 3.4: Every g-binary θ-pre-closed set (g-binary θ-β-closed) is g-binary closed but not converse as shown in Example 3.4 and Example 3.5.

Example 3.4: In Example 3.1 clearly the set $(\{1\}, \{a, b\})$ is g-binary closed but not g-binary θ-pre-closed because $gbcl(gbint_\theta(\{1\}, \{a, b\})) \not\subseteq (\{1\}, \{a, b\})$.

Example 3.5: In Example 3.1 clearly the set $(\{1\}, \{a, b\})$ is g-binary closed but not g-binary θ-β-closed because $gbint(gbcl(gbint_\theta(\{1\}, \{a, b\}))) \not\subseteq (\{1\}, \{a, b\})$.

Remark 3.5: Every g-binary θ-pre-closed is g-binary θ-closed and but not converse as shown in Example 3.6.

Example 3.6: In Example 3.1 the set $(\{1\}, \{a, b\})$ is g-binary θ-closed but not g-binary θ-pre closed.

Remark 3.6: Every g-binary θ-β-closed set is g-binary θ-closed but not converse as shown in Example 3.7.

Example 3.7: In Example 3.1 the set $(\{1\}, \{a, b\})$ is g-binary θ-β-open but not g-binary θ-open.

From above discussion we conclude that:

- *g-binary closed* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-closed*
- *g-binary closed* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-semi-closed*
- *g-binary closed* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-*α*-closed*
- *g-binary* θ*-pre-closed* $\Rightarrow (\not\Leftarrow)$ *g-binary closed*
- *g-binary* θ*-*β*-closed* $\Rightarrow (\not\Leftarrow)$ *g-binary closed*
- *g-binary* θ*-pre-closed* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-closed*
- *g-binary* θ*-*β*-closed* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-closed*

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4. g-BINARY θ-CONTINUOUS MAPS

Definition 4.1: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. The map $f: Z \to X \times Y$ is called g-binary θ-continuous if $f^{-1}(A, B)$ is g-θ-open (g-θ-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 4.2: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. The map $f: Z \to X \times Y$ is called g-binary θ-semi-continuous if $f^{-1}(A, B)$ is g-θ-semi-open (g-θ-semi-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 4.3: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. The map $f: Z \to X \times Y$ is called g-binary θ-pre-continuous if $f^{-1}(A, B)$ is g-θ-pre open (g-θ-pre-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 4.4: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. The map $f: Z \to X \times Y$ is called g-binary θ-α-continuous if $f^{-1}(A, B)$ is g-θ-α-open (g-θ-α-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Definition 4.5: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. The map $f: Z \to X \times Y$ is called g-binary θ-β-continuous if $f^{-1}(A, B)$ is g-θ-β-open (g-θ-β-closed) in (Z, τ) for every g-binary open (g-binary closed) set (A, B) in (X, Y, M_g).

Proposition 4.1: Every g-binary continuous map in g-binary topology is g-binary θ-continuous.

Proof: Let (A, B) be a g-binary closed set in (X, Y, M_g). Since f is g-binary continuous, we have $f^{-1}(A, B)$ is g-closed in (Z, τ). We know that every g-closed set is g-θ-closed. Hence $f^{-1}(A, B)$ is g-θ-closed in (Z, τ). Thus f is g-binary θ-semi-continuous map.

Remark 4.1: The converse of the Proposition 4.1 need not be true as shown in Example 4.1.

Example 4.1: Let $Z = \{1, 2, 3\}$, $X = \{a_1, a_2\}$ and $Y = \{b_1, b_2\}$. Then $\tau = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_1\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1) = f(3)$ and $f(2) = (a_2, b_2)$. Thus the inverse image of every g-binary closed set in (X, Y, M_g) is g-θ--closed in (Z, τ). Hence f is g-binary θ-continuous but not g-binary continuous because $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{2\}$, where $\{1,3\}$ and $\{2\}$ are g-θ-closed but not g-closed in (Z, τ).

Proposition 4.2: Every g-binary θ-pre-continuous in g-binary topology is g-binary θ-continuous.

Proof: Let (A, B) be a g-binary closed set in (X, Y, M_g). Since f is g-binary θ-pre-continuous, we have $f^{-1}(A, B)$ is g-θ-pre-closed in (Z, τ). We know that every g-θ-pre-closed set is g-θ-closed. Hence $f^{-1}(A, B)$ is g-θ-pre-closed in (Z, τ). Thus f is g-binary θ-pre-continuous map.

Remark 4.2: The converse of the Proposition 4.2 need not be true as shown in Example 4.2.

Example 4.2: In Example 4.1 f is g-binary θ-continuous but not g-binary θ-pre-continuous because $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{2\}$, where $\{1,3\}$ and $\{2\}$ are g-θ-closed but not g-θ-pre-closed in (Z, τ).

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Proposition 4.3: Every g-binary θ-β-continuous in g-binary topology is g-binary θ-continuous.

Proof: Quite easy.

Remark 4.3: The converse of the Proposition 4.3 need not be true as shown in Example 4.3.

Example 4.3: In Example 4.1 f is g-binary θ-continuous but not g-binary θ-pre-continuous because $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{2\}$, where $\{1,3\}$ and $\{2\}$ are g-θ-closed but not g-θ-β-closed in (Z, τ).

Proposition 4.4: Every g-binary continuous in g-binary topology is g-binary θ-semi-continuous.

Proof: Quite easy

Remark 4.4: The converse of the Proposition 4.4 need not be true as shown in Example 4.4.

Example 4.4: In Example 4.1 f is g-binary θ-semi-continuous but not g-binary continuous because $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{2\}$, where $\{1,3\}$ and $\{2\}$ are g-θ-semi-closed but not g-closed in (Z, τ).

Proposition 4.5: Every g-binary continuous in g-binary topology is g-binary θ-α-continous.

Proof: Quite easy

Remark 4.5: The converse of the Proposition 4.5 need not be true as shown in Example 4.5.

Example 4.5: In Example 4.1 f is g-binary θ - α -continuous but not g-binary continuous because $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{2\}$, where $\{1,3\}$ and $\{2\}$ are g-θ-α-closed but not g-closed in (Z, τ).

Proposition 4.6: Every g-binary θ-pre-continuous in g-binary topology is g-binary continuous.

Proof: Quite easy

Remark 4.6: The converse of the Proposition 4.6 need not be true as shown in Example 4.6.

Example 4.6: In Example 4.1 define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1)$, $f(2) = (\emptyset, b_2)$ and $f(3) = (a_2, b_2)$. Thus the inverse image of every g-binary closed set in (X, Y, M_g) is g-closed in (Z, τ). Hence f is g-binary continuous but not g-binary θ-pre-continuous, because $f^{-1}(\{a_1\}, \{b_1\}) = \{1\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{3\}$, where $\{1\}$ and $\{3\}$ are g-closed but not g-θ-pre-closed in (Z, τ).

Proposition 4.7: Every g-binary θ-β-continuous in g-binary topology is g-binary continuous.

Proof: Quite easy.

Remark 4.7: The converse of the Proposition 4.7 need not be true as shown in Example 4.7.

Example 4.7: In Example 4.1 define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1)$, $f(2) = (\emptyset, b_2)$ and $f(3) = (a_2, b_2)$. Thus the inverse image of every g-binary closed set in (X, Y, M_g) is g-closed in (Z, τ). Hence f is g-binary continuous but not g-binary θ-β-continuous, because $f^{-1}(\{a_1\}, \{b_1\}) = \{1\}$ and $f^{-1}(\{a_2\}, \{b_2\}) = \{3\}$, where $\{1\}$ and $\{3\}$ are g-closed but not g-θ-β-closed in (Z, τ).

From above discussion we conclude that:

- *g-binary continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-continuous*
- *g-binary* θ*-pre-continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-continuous*
- *g-binary* θ*-*β*-continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-continuous*

- *g-binary continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-semi-continuous*
- *g-binary continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary* θ*-*α*-continuous*
- *g-binary* θ*-pre-continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary continuous*
- *g-binary* θ*-*β*-continuous* $\Rightarrow (\not\Leftarrow)$ *g-binary continuous*

CONCLUSION

The concept of g-binary θ-closed sets and g-binary θ-continuous maps in g-binary topological spaces have been introduced and studied. Further relationships have also been established between these sets and maps.

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