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BAYESIAN ESTIMATION OF SHAPE AND SCALEPARAMETERS OF ERLANG DISTRIBUTION

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ABSTRACT

Erlang distribution is a continuous probability distribution with wide applicability primarily due to its relation to the exponential and Gamma distributions. In this paper Bayesian estimation of shape and scale parameters of Erlang distribution have been obtained using various Size biased Poisson, Consul, Geeta distribution. Various special cases have also been discussed.

Key words*: *Erlang distribution, informative prior, loss function, Size biased distributions, Gamma distribution.

1. INTRODUCTION

Erlang distribution is a continuous probability distribution, which has a positive value for all real numbers greater than zero, and is given by two parameters: the shape parameter 'u', which is a positive integer, and the rate parameter 'q', which is a positive real number. The distribution is sometimes defined in terms ofthe scale parameter 'v', the inverse of the rate parameter. Eralang distribution is the distribution of the sum of u independent exponential random variables with mean v. When the shape parameter u equals 1, the distribution simplifies to the exponential distribution. The Erlang distribution is a special case of the Gamma distribution when the shape parameter u is an integer (Evans et. al. 2000). To examine the number of telephone calls which might be made at the same time to the operators of the switching stations A. K. Erlang introduced the Erlang distribution. This work on telephone traffic engineering has been expanded to consider waiting times in queuing systems in general. The queuing theory had its origin in 1909, when A.K. Erlang (1878-1929) presented his fundamental paper regarding"The Theory of Probabilities and Telephone Conversations"proving that telephone calls distributed at random follow Poisson's law of distribution. Further, the most important work was published in 1917 "Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges". This paper contained formulae for loss and waiting time, which are now well known in the theory of telephone traffic. A comprehensive survey of his works is given in "The life and works of A.K. Erlang".

The probability function of Erlang distribution is given by

$$f(x{:}\,u,v) = \frac{x^{u-1}\exp(-v^{-1}x)}{\Gamma(u)v^{u}};\ u = 1,2,3,\ldots\ldots;\ v > 0,\ x > 0 \qquad (1.1)$$

Where, “u” and “v” are called shape and scale parameter respectively.

And in terms of rate parameter $q(= {}^{1}/_{v})$, Erlang distribution is given by

$$f(x{:}\,u,q) = \frac{q^{u}x^{u-1}\exp(-qx)}{\Gamma(u)};\ u = 1,2,3,\ldots\ldots;\ q = {}^{1}/_{v} > 0,\ x > 0 \qquad (1.2)$$

Where, q is called rate parameter, when the scale parameter $v = 2$, the distribution is the chi-squared distribution with 2k degrees of freedom. Therefore, it can be regarded as a generalized chi-squared distribution. Under two prior densities Bhattacharyya and Singh (1994) obtained Bayes estimator for the Erlangian Queue. Jain(2001) discussed the problem of the change point for the inter arrival time distribution in the context of exponential families for the Ek/G/e queuing system and obtained Bayes estimate of the posterior probabilities and the position of the change from the Erlang distribution. Nair et al (2003) studies Erlang distribution as a model for ocean wave periods and obtained different characteristics of this distribution under classical set up. Suri et al (2009) designed a simulator for time estimation of project management process using Erlang distribution. Damodaran et. al. (2010) showed that the actual failure times are closer to the predicted failure time and obtained the expected time between failure measures. Ab. Haq and Sanku Dey (2011) addressed the problem of Bayesian estimation of parameters of Erlang distribution assuming different independent informative priors. In this paper parameters of Erlang distribution are estimated using Consul, Geeta and Size Biased Poisson distributions.

2. PRIOR AND LOSS FUNCTION

The information is always available in an independent manner about the shape and scale parameters of the sampling distribution. Therefore, a number of prior distributions have been taken into consideration and it is assumed that the parameters u and v are independent. These are:

(a) Consul distribution as a prior for shape parameter.
(b) Geeta distribution as a prior for shape parameters.
(c) Size Biased Poisson distribution as a prior for shape parameters.
(d) Consul and Inverted Gamma priors for shape and scale parameter.
(e) Consul and Gamma priors for shape and scale parameter.
(f) Geeta and Inverted Gamma priors for shape and scale parameter.
(g) Geeta and Gamma priors for shape and scale parameter.
(h) Size Biased Poisson and Inverted Gamma priors for shape and scale parameter.

(i) Size Biased Poisson and Gamma priors for shape and scale parameter.

The loss function considered in this paper is squared error loss function. The squared error loss function for the shape parameters "u" and the scale parameters "v" are defined as

$$L(\hat{u}) = (\hat{u} - u)^2$$

$$L(\hat{v}) = (\hat{v} - v)^2$$

which is symmetric. u, v and $\hat{u}$ and $\hat{v}$ represent the true and estimated values of the parameters.

3. DERIVATION OF POSTERIOR DISTRIBUTION UNDER DIFFERENT INFORMATIVE PRIORS

Let X_1, X_2, X_3, …,X_n be a random sample from Erlang distribution, the likelihood function of the sample observations x_1, x_2, x_3,…,x_n is defined as:

$$L(u, v{:}x) = \frac{\prod_{i=1}^{n} x_i^{u-1} \exp\left(-\frac{1}{v}\sum_{i=1}^{n} x_i\right)}{\left(\Gamma(u)\right)^n v^{nu}}; \quad u = 1,2,3, \dots; v > 0 \qquad (3.1)$$

3.1 When shape parameter "u" is unknown and scale parameter "v" is known

It is known the performance of the Bayes estimators depends on the form of the prior distribution and the loss function assumed. In this section, we assume three different informative prior distributions for the shape parameter u, viz., Consul Distribution, Geeta Distribution and Size Biased Poisson Distribution and obtain the Bayes estimators and posterior variances. Also, Bayes estimators and posterior variances are also obtained for Haights distribution and truncated geometric distribution as the special cases.

3.1.1 Consul distribution as a prior for shape parameter

The pdf of Consul Distribution is

$$g_1(u, \theta_1) = \frac{1}{u}\binom{mu}{u-1}\theta_1^{u-1}(1-\theta_1)^{mu-u+1};$$

$$u = 1,2,3, \dots, 0 \le \theta_1 < 1; 1 \le m \le \frac{1}{\theta_1} \qquad (3.1.1.1)$$

By combining the likelihood function (3.1) and the prior density function (3.1.1.1), the posterior distribution of u, when prior is Consul Distribution, is given by

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$$g_1(u/X) = \frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}$$

Under squared error loss function with the prior$g_1(u, \theta_1)$, the Bayes estimator is

$$\hat{u}_1 / X = \frac{\sum_{u=1}^{\infty}\left(\frac{u\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}$$

$$\hat{u}_1 / X = \frac{\sum_{u=1}^{\infty}\left(\frac{\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}$$

The posterior variance of Bayes estimator $\hat{u}_1/_X$ is given by

$$\mathbf{Var}(\hat{u}_1 / X) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u^2\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}\right) - (\hat{u}_1 / X)^2$$

$$\mathbf{Var}(\hat{u}_1 / X) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}\right) - (\hat{u}_1 / X)^2$$

Special cases

For $m = 1$ Consul Distribution tends to truncated geometric distribution; therefore, the Bayes estimator for scale parameter "u" is given by

$$\hat{u}_{ge1} / X = \frac{\sum_{u=1}^{\infty}\left(\frac{u\theta_1^u\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp(u\sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}}\right)}$$

and posterior variance of Bayes estimator $\hat{u}_{ge1} / X$ is given by

$$\mathrm{Var}(\hat{u}_{ge1} / X) = \left(\frac{\sum_{u=1}^{\infty} \left(\frac{u^2 \theta_1^u \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\theta_1^u \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)} \right) - (\hat{u}_1 / X)^2$$

(See Abdul Haq and Sanku Dey, 2011)

3.1.2 Geeta distribution as a prior for shape parameter

This time the prior assumed for shape parameter is Geeta distribution and the pdf of Geeta distribution is given by

$$g_2(u, \theta_2) = \frac{1}{\beta u - 1} \binom{\beta u - 1}{u} \theta_2^{u-1} (1 - \theta_1)^{\beta u - u} ;$$

$$u = 1,2,3, \ldots ; 0 \leq \theta_2 < 1 , 1 < \beta < \theta^{-1} \qquad (3.1.2.1)$$

By combining the likelihood function (3.1) and the prior density function (3.1.2.1), the posterior distribution of u, when the prior is Geeta distribution, is given by

$$g_2(u/X) = \frac{\frac{\frac{1}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}}}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}$$

Under squared error loss function with the prior $g_2(u, \theta_2)$, the Bayes estimator is

$$\hat{u}_2 / X = \frac{\sum_{u=1}^{\infty} \left(\frac{\frac{u}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}$$

$$\hat{u}_2 / X = \frac{\sum_{u=1}^{\infty} \left(\frac{\binom{\beta u - 2}{u - 1} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}$$

and posterior variance of Bayes estimator $\hat{u}_2/_X$ is given by

$$\mathrm{Var}(\hat{u}_2 / X) = \left(\frac{\sum_{u=1}^{\infty} \left(\frac{\frac{u^2}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{\beta u - 1}\binom{\beta u - 1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^{n} \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)} \right) - (\hat{u}_2 / X)^2$$

$$\mathrm{Var}(\hat{u}_2 / X) = \left(\frac{\sum_{u=1}^{\infty} \left(\frac{u\binom{\beta u-2}{u-1} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u} \theta_2^u (1-\theta_1)^{\beta u - u} \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)} \right) - (\hat{u}_2 / X)^2$$

Special cases

For $\beta = 2$ Geeta Distribution reduces to Haight distribution, therefore the Bayes estimator for scale parameter u is given by

$$\hat{u}_{ha2} / X = \frac{\sum_{u=1}^{\infty} \left(\frac{\binom{2(u-1)}{u-1} \theta_2^u (1-\theta_1)^u \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{2u-1}\binom{2u-1}{u} \theta_2^u (1-\theta_1)^u \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}$$

and posterior variance of Bayes estimator $\hat{u}_{ha2} / X$ is given by

$$Var(\hat{u}_{ha2} / X) = \left(\frac{\sum_{u=1}^{\infty} \left(\frac{u\binom{2(u-1)}{u-1} \theta_2^u (1-\theta_1)^u \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)}{\sum_{u=1}^{\infty} \left(\frac{\frac{1}{2u-1}\binom{2u-1}{u} \theta_2^u (1-\theta_1)^u \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^n v^{nu}} \right)} \right) - (\hat{u}_{ha2} / X)^2$$

3.1.3 Size Biased Poisson distribution as a prior for shape parameter

This time the prior assumed for shape parameter is Size Biased Poisson distribution. And the pdf of Size Biased Poisson distribution is given by

$$g_3(u, \theta_3) = \frac{e^{-\theta_3} \theta_3^{u-1}}{(u-1)!} \; ; \theta > 0, u = 1,2,3, \ldots (3.1.3.1)$$

$$or\, g_3(u, \theta_3) = \frac{e^{-\theta_3} \theta_3^{u-1}}{\Gamma(u)} \; ; \theta > 0, u = 1,2,3, \ldots \; (3.1.3.2)$$

By combining the likelihood function (3.1) and the prior density function (3.1.3.2), the posterior distribution of u, when prior is Size Baised Poisson Distribution, is given by

$$g_3(u/x) = \frac{\frac{\theta_3^u \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^{n+1} v^{nu}}}{\sum_{u=1}^{\infty} \left(\frac{\theta_3^u \exp(u \sum_{i=1}^n \ln x_i)}{(\Gamma(u))^{n+1} v^{nu}} \right)}$$

Under squared error loss function with the prior $g_3(u, \theta_3)$, the Bayes estimator is

$$\hat{u}_3/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{u\theta_3^u exp\,(u\sum_{i=1}^{n} ln\, x_i)}{(\Gamma(u))^{n+1} v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\,(u\sum_{i=1}^{n} ln\, x_i)}{(\Gamma(u))^{n+1} v^{nu}}\right)}$$

The posterior variance of Bayes estimator $\hat{u}_3/_X$ is given by

$$Var\left(\hat{u}_3/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u^2\theta_3^u exp\,(u\sum_{i=1}^{n} ln\, x_i)}{(\Gamma(u))^{n+1} v^{nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\,(u\sum_{i=1}^{n} ln\, x_i)}{(\Gamma(u))^{n+1} v^{nu}}\right)}\right) - \left(\hat{u}_3/_X\right)^2$$

3.2 When both shape and scale parameters are unknown

The different independent prior distributions are assumed for two unknown parameters u (shape) and v (scale) of Erlang distributions. The Bayes estimators and posterior variances for both shape parameter u and scale parameter v of Erlang distribution are derived in the following section.

3.2.1 Posterior Distribution under Consul and Inverted Gamma priors

The assumed prior for the shape parameter u of the Erlang distribution is Consul Distribution having pdf

$$g_1(u,\theta_1) = \frac{1}{u}\binom{mu}{u-1}\theta_1^{u-1}(1-\theta_1)^{mu-u+1}\ ; u = 1,2,3,\dots, 0 \le \theta_1 < 1; 1 \le m \le \frac{1}{\theta_1}$$

And the prior for scale parameter v is assumed to be Inverted Gamma distribution having pdf

$$g_4(v;\alpha_1,\beta_1) = \frac{\beta_1^{\alpha_1} v^{-(\alpha_1+1)} exp(-v^{-1}\beta_1)}{\Gamma(\alpha_1)}\ ; v > 1, \alpha_1 > 0, \beta_1 > 0$$

The joint Prior of u and v is defined as:

$$g_{14}(u,v) \propto g_1(u,\theta_1).\, g_4(v;\alpha_2,\beta_2) \qquad (3.2.1.1)$$

Combining the likelihood function (3.1) and the joint density (3.2.1.1), the joint posterior distributions of u and v are given by

$$g_{14}(u, v/_X) = \frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u} exp\left(u\sum_{i=1}^{n} ln\, x_i\right) v^{-(\alpha_1+nu+1)} exp\left\{-v^{-1}(\beta_1+\sum_{i=1}^{n} x_i)\right\}}{(\Gamma(u))^n}}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u} exp\left(u\sum_{i=1}^{n} ln\, x_i\right)\int_o^{\infty} v^{-(\alpha_1+nu+1)} exp\left\{-v^{-1}(\beta_1+\sum_{i=1}^{n} x_i)\right\} dv}{(\Gamma(u))^n}\right)}$$

$$= \frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)v^{-(\alpha_1+nu+1)}exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i)\}}{(\Gamma(\omega))^n}}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

The marginal posterior distribution of u and v are:

$$g_{14}\left({}^{u}/_{X}\right) = \frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\int_0^{\infty}v^{-(\alpha_1+nu+1)}exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i)\}dv}{(\Gamma(\omega))^n}}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$= \frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$g_{14}\left({}^{v}/_{X}\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)v^{-nu}}{(\Gamma(\omega))^n}\right)\left(\frac{exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i\}}{v^{\alpha_1+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

The expression for Bayes estimators of u and v under squared error loss function with their respective Posterior variance are given below:

$$\hat{u}_{14}/_{X} = \frac{\sum_{u=1}^{\infty}\left(\frac{\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

and,

$$\hat{v}_{14}/_{X} = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\int_0^{\infty}v.v^{-(\alpha_1+nu+1)}exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i)\}dv}{(\Gamma(\omega))^n}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$\hat{v}_{14}/_{X} = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu-1)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu-1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^n lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^n(\beta_1+\sum_{i=1}^n x_i)^{\alpha_1+nu}}\right)}$$

also,

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$$Var\left(\hat{u}_{14}/_X\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{u\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)} - \left(\hat{u}_{14}/_X\right)^2$$

and,

$$Var\left(\hat{v}_{14}/_X\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\int_0^{\infty}v^2v^{-(\alpha_1+nu+1)}exp\left\{-v^{-1}\left(\beta_1+\sum_{i=1}^{n}x_i\right)\right\}db}{\left(\Gamma(w)\right)^n}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}$$

$$-\left(\hat{v}_{14}/_X\right)^2$$

$$Var\left(\hat{v}_{14}/_X\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu-2)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu-2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)} - \left(\hat{v}_{14}/_X\right)^2$$

Special cases

For $m = 1$ Consul Distribution tends to truncated geometric distribution; therefore, the Bayes estimator for scale parameter u and shape parameter v are given by

$$\hat{u}_{14}{}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{u\theta_1^{u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^{u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}$$

and,

$$\hat{v}_{14}{}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_1^{u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu-1)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu-1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^{u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}$$

Also, respective Posterior variance of u and v are given below

$$Var\left(\hat{u}_{14}{}^{*}/_X\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{u^2\theta_1^{u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^{u}exp\left(u\sum_{i=1}^{n}lnx_i\right)\Gamma(\alpha_1+nu)}{\left(\Gamma(w)\right)^n\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)} - \left(\hat{u}_{14}{}^{*}/_X\right)^2$$

and,

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$$Var\left({\hat{v}_{14}}^{*}/X\right)=\frac{\sum_{u=1}^{\infty}\left(\frac{\theta_1^{u}exp\left(u\sum_{i=1}^{n}ln\,x_i\right)\Gamma(\alpha_1+nu-2)}{(\Gamma(\omega))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu-2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^{u}exp\left(u\sum_{i=1}^{n}ln\,x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}-\left({\hat{v}_{14}}^{*}/X\right)^{2}$$

(See Abdul Haq and Sanku Dey, 2011)

3.2.2 Posterior Distribution under consul and Gamma Priors

Again the prior for u is assumed to Consul Distribution and for v the assumed prior is Gamma Distribution, having the following pdf

$$g_1(u,\theta_1)=\frac{1}{u}\binom{mu}{u-1}\theta_1^{u-1}(1-\theta_1)^{mu-u+1}\,;\,u=1,2,3,\ldots,0\le\theta_1<1;1\le m\le\frac{1}{\theta_1}$$

and,

$$g_5(v;\alpha_2,\beta_2)=\frac{\beta_2^{\alpha_2}v^{\alpha_2-1}exp(-v\beta_2)}{\Gamma(\alpha_2)}\quad;\qquad v>0,\alpha_2>0,\beta_2>0$$

The joint Prior Distribution of u and v is defined as:

$$g_{15}(u,v)\propto g_1(u,\theta_1).\,g_5(v;\alpha_2,\beta_2) \qquad (3.2.2.1)$$

Combining the likelihood function (3.1) and the joint density (3.2.2.1), the joint posterior distributions of u and v are given by

$$g_{15}\left(u,{}^{v}/_{X}\right)=\frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{\left(2\sum_{i=1}^{n}x_i\right)}{v}\right)\right\}}{exp\left(-u\sum_{i=1}^{n}ln\,x_i\right)(\Gamma(\omega))^{n}}}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}\int_{o}^{\infty}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{\left(2\sum_{i=1}^{n}x_i\right)}{v}\right)\right\}dv}{exp\left(-u\sum_{i=1}^{n}ln\,x_i\right)(\Gamma(\omega))^{n}}\right)}$$

$$g_{15}\left(u,{}^{v}/_{X}\right)=\frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{\left(2\sum_{i=1}^{n}x_i\right)}{v}\right)\right\}}{exp\left(-u\sum_{i=1}^{n}ln\,x_i\right)(\Gamma(\omega))^{n}}}{\sum_{u=1}^{\infty}\left(\frac{\frac{2}{u}\binom{mu}{u-1}\theta_1^{u}(1-\theta_1)^{mu-u}exp\left(u\sum_{i=1}^{n}ln\,x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2\left(\sum x_i\right)}\right)}{(\Gamma(\omega))^{n}\left(\sqrt{\left(\sum x_i\right)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

Here, $K_t(.)$ is modified Bessel function of third kind with index t.

The marginal posterior distribution of u and v are:

$$g_{15}\left({}^{u}/_{X}\right)=\frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}}{(\Gamma(\omega))^n exp(-u\sum_{i=1}^n lnx_i)}\int_0^\infty v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv}{\sum_{u=1}^\infty\left(\frac{\frac{2}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$g_{15}\left({}^{u}/_{X}\right)=\frac{\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n exp(-u\sum_{i=1}^n lnx_i)\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}}{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$g_{15}\left({}^{v}/_{X}\right)=\frac{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}}{(\Gamma(\omega))^n exp(-u\sum_{i=1}^n lnx_i)}\right)}{\sum_{u=1}^\infty\left(\frac{\frac{2}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

The expression for Bayes estimators of u and v with their respective Posterior variance are given below:

$$\hat{u}_{15}/_{X}=\frac{\sum_{u=1}^\infty\left(\frac{\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\,exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

$$\hat{v}_{15}/_{X}=\frac{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}}{(\Gamma(\omega))^n exp(-u\sum_{i=1}^n lnx_i)}\int_0^\infty v.\,v^{(\alpha_2-nu)-1}\,exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv\right)}{\sum_{u=1}^\infty\left(\frac{\frac{2}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$\hat{v}_{15}/_{X}=\frac{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\,exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu+1}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+1}}\right)}{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

$$Var\left({}^{\hat{u}_{15}}/_{X}\right)=\left(\frac{\sum_{u=1}^\infty\left(\frac{u\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\,exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^\infty\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(\omega))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right)-\left({}^{\hat{u}_{15}}/_{X}\right)^2$$

$$Var\left(\hat{v}_{15}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu+2}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{u}\binom{mu}{u-1}\theta_1^u(1-\theta_1)^{mu-u}\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left(\hat{v}_{15}/_X\right)^2$$

Special cases:

For $m = 1$ Consul Distribution tends to truncated geometric distribution; therefore, the Bayes estimator for scale parameter u and shape parameter v are given by

$$\hat{u}_{15}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{u\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

$$\hat{v}_{15}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu+1}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

Also, respective Posterior variance of u and v are given below

$$Var\left(\hat{u}_{15}^{*}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u^2\,\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left(\hat{u}_{15}^{*}/_X\right)^2$$

$$Var\left(\hat{v}_{15}^{*}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu+2}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_1^u\exp\left(u\sum_{i=1}^{n}\ln x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left(\hat{v}_{15}^{*}/_X\right)^2$$

3.2.3 Posterior distribution under Geeta and Inverted Gamma distribution Priors

The prior assumed for shape parameter u is Geeta distribution and pdf is given by

$$g_2(u,\theta_2) = \frac{1}{\beta u - 1}\binom{\beta u - 1}{u}\theta_2^{u-1}(1-\theta_2)^{\beta u - u}\,;\, u = 1,2,3,\ldots;\;\; 0 \le \theta_2 < 1\,, 1 < \beta < \theta_2^{-1}$$

And, for the scale parameter v the assumed prior is Inverted Gamma distribution having pdf

$$g_4(v;\alpha_1,\beta_1) = \frac{\beta_1^{\alpha_1} v^{-(\alpha_1+1)} exp(-v^{-1}\beta_1)}{\Gamma(\alpha_1)} \; ; v > 1, \alpha_1 > 0, \beta_1 > 0$$

The joint Prior Distribution of u and v is defined as:

$$g_{24}(u,v) \propto g_2(u,\theta_2).\, g_4(v;\alpha_1,\beta_1) \qquad (3.2.3.1)$$

By combining the likelihood function (3.1) and the joint prior function (3.2.3.1), the joint posterior distribution of u and v is given by

$$g_{24}\left(u,\frac{v}{X}\right) = \frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)v^{-(\alpha_1+nu+1)} exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i\}}{(\Gamma(u))^n}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\int_o^{\infty} v^{-(\alpha_1+nu+1)} exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i)\}dv}{(\Gamma(u))^n}\right)}$$

$$g_{24}\left(u,\frac{v}{X}\right) = \frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)v^{-(\alpha_1+nu+1)} exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i\}}{(\Gamma(u))^n}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

The marginal posterior distribution of u and v are given below

$$g_{24}\left(\frac{u}{X}\right) = \frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\int_o^{\infty} v^{-(\alpha_1+nu+1)} exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i)\}dv}{(\Gamma(u))^n}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$g_{24}\left(\frac{u}{X}\right) = \frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

and,

$$g_{24}\left(\frac{v}{X}\right) = \frac{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)v^{-nu}}{(\Gamma(u))^n}\left(\dfrac{exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i\}}{v^{\alpha_1+1}}\right)\right)}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

Under squared loss function, the expression for Bayes estimators of u and v with their respective Posterior variance are given below

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$$\hat{u}_{24}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\binom{\beta u-2}{u-1}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$\hat{v}_{24}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)}{(\Gamma(u))^n}\left(\int_0^{\infty}\frac{v.exp\{-v^{-1}(\beta_1+\sum_{i=1}^n x_i\}}{v^{\alpha_1+nu+1}}dv\right)\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$\hat{v}_{24}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu-1)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu-1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

Also,

$$Var\left(\hat{u}_{24}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u\binom{\beta u-2}{u-1}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}\right) - \left(\hat{u}_{24}/_X\right)^2$$

and

$$Var\left(\hat{v}_{24}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu-2)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu-2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}\right) - \left(\hat{v}_{24}/_X\right)^2$$

Special cases

For $\beta = 2$ Geeta Distribution reduces to Haight distribution; therefore, the Bayes estimator for scale parameter u and shape parameter v are given by

$$\hat{u}_{24}{}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\binom{2(u-1)}{u-1}\theta_2^u(1-\theta_2)^{u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^{u}exp(u\sum_{i=1}^n lnx_i)\Gamma(\alpha_1+nu)}{(\Gamma(u))^n(\beta_1+\sum_{i-1}^n x_i)^{\alpha_1+nu}}\right)}$$

$$\hat{v}_{24}^{*}/_{X} = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^{u}(1-\theta_2)^{u}u\,exp\left(u\sum_{i=1}^{n}\ln x_i\right)\Gamma(\alpha_1+nu-1)}{(\Gamma(u))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu-1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^{u}(1-\theta_2)^{u}exp\left(u\sum_{i=1}^{n}\ln x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}$$

Also, respective Posterior variance of u and v are given below

$$Var\left(\hat{u}_{24}^{*}/_{X}\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u\binom{2(u-1)}{u-1}\theta_2^{u}(1-\theta_2)^{u}exp\left(u\sum_{i=1}^{n}\ln x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^{u}(1-\theta_2)^{u}exp\left(u\sum_{i=1}^{n}\ln x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}\right) - \left(\hat{u}_{24}^{*}/_{X}\right)^2$$

and,

$$Var\left(\hat{v}_{24}^{*}/_{X}\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^{u}(1-\theta_2)^{u}exp\left(u\sum_{i=1}^{n}\ln x_i\right)\Gamma(\alpha_1+nu-2)}{(\Gamma(u))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu-2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^{u}(1-\theta_2)^{u}exp\left(u\sum_{i=1}^{n}\ln x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n}\left(\beta_1+\sum_{i=1}^{n}x_i\right)^{\alpha_1+nu}}\right)}\right) - \left(\hat{v}_{24}^{*}/_{X}\right)^2$$

3.2.4 Posterior distribution under Geeta and Gamma Priors

The prior assumed for shape parameter u is Geeta distribution and pdf is given by

$$g_2(u,\theta_2) = \frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^{u-1}(1-\theta_2)^{\beta u-u}\,;\, u = 1,2,3,\ldots;\, 0 < \theta_2 < 1\,,\, 1 < \beta < \theta_2^{-1}$$

And, for the scale parameter v the assumed prior is Gamma distribution having pdf

$$g_5(v;\alpha_2,\beta_2) = \frac{\beta_2^{\alpha_2}v^{\alpha_2-1}exp(-v\beta_2)}{\Gamma(\alpha_2)}\,;\, v > 1, \alpha_2 > 0, \beta_2 > 0$$

The joint Prior Distribution of u and v is defined as:

$$g_{25}(u,v) \propto g_2(u,\theta_2).\,g_5(v;\alpha_2,\beta_2) \qquad (3.2.4.1)$$

By combining the likelihood function (3.1) and the joint prior function (3.2.4.1), the joint posterior distribution of u and v is given by

$$g_{25}\left(u,{}^{v}/_{X}\right)=\frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}}{exp(-u\sum_{i=1}^n ln\,x_i)(\Gamma(u))^n}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}\int_0^{\infty}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv}{exp(-u\sum_{i=1}^n ln\,x_i)(\Gamma(u))^n}\right)}$$

$$g_{25}\left(u,{}^{v}/_{X}\right)=\frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}}{exp(-u\sum_{i=1}^n ln\,x_i)(\Gamma(u))^n}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{2}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

The marginal posterior distribution of u and v are:

$$g_{25}\left({}^{u}/_{X}\right)=\frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}}{(\Gamma(u))^n exp(-u\sum_{i=1}^n ln\,x_i)}\displaystyle\int_0^{\infty}v^{(\alpha_2-nu)-1}\,exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{2}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$g_{25}\left({}^{u}/_{X}\right)=\frac{\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n exp(-u\sum_{i=1}^n ln\,x_i)\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$g_{25}\left({}^{v}/_{X}\right)=\frac{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}}{(\Gamma(u))^n exp(-u\sum_{i=1}^n ln\,x_i)}\right)}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{2}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

The expression for Bayes estimators of u and v with their respective Posterior variance are given below:

$$\hat{u}_{25}/_{X}=\frac{\sum_{u=1}^{\infty}\left(\dfrac{\binom{\beta u-2}{u-1}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^{\infty}\left(\dfrac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}exp(u\sum_{i=1}^n ln\,x_i)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

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$$\hat{v}_{25}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u}}{(\Gamma(u))^n exp(-u\sum_{i=1}^n lnx_i)}\int_0^{\infty} v.\, v^{(\alpha_2-nu)-1}\, exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{2}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$\hat{v}_{25}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu+1}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

$$Var\left(\hat{u}_{25}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u\binom{\beta u-2}{u-1}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left(\hat{u}_{25}/_X\right)^2$$

$$Var\left(\hat{v}_{25}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu+2}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{\beta u-1}\binom{\beta u-1}{u}\theta_2^u(1-\theta_2)^{\beta u-u} exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left(\hat{v}_{25}/_X\right)^2$$

Special cases

For $\beta = 2$ Geeta Distribution reduces to Haight distribution; therefore, the Bayes estimator for scale parameter u and shape parameter v are given by

$$\hat{u}_{25}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\binom{2(u-1)}{u-1}\theta_2^u(1-\theta_2)^u exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^u exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$\hat{v}_{25}^{*}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^u exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu+1}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^u exp(u\sum_{i=1}^n lnx_i)K_{\alpha_2-nu}(2\sqrt{\beta_2(\sum x_i)})}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

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$$Var\left({\hat{u}_{25}}^{*}/_{X}\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u\binom{2(u-1)}{u-1}\theta_2^u(1-\theta_2)^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left({\hat{u}_{25}}^{*}/_{X}\right)^2$$

$$Var\left({\hat{v}_{25}}^{*}/_{X}\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)K_{\alpha_2-nu+2}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\frac{1}{2u-1}\binom{2u-1}{u}\theta_2^u(1-\theta_2)^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^n\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}\right) - \left({\hat{v}_{25}}^{*}/_{X}\right)^2$$

3.2.5 Posterior Distribution under Size Biased Poisson and Inverted Gamma priors

The assumed prior for the shape parameter u of the Erlang distribution is Size Biased Poisson distribution having pdf

$$g_3(u,\theta_3) = \frac{e^{-\theta_3}\theta_3^{u-1}}{\Gamma(u)}\ ;\ \theta_3 > 0, u = 1,2,3,\ldots$$

And, for the scale parameter v the assumed prior is Inverted Gamma distribution having pdf

$$g_4(v;\alpha_1,\beta_1) = \frac{\beta_1^{\alpha_1} v^{-(\alpha_1+1)} exp(-v^{-1}\beta_1)}{\Gamma(\alpha_1)}\ ; v > 0, \alpha_1 > 0, \beta_1 > 0$$

The joint Prior Distribution of u and v is defined as:

$$g_{34}(u,v) \propto g_3(u,\theta_3).\, g_4(v;\alpha_1,\beta_1) \qquad (3.2.5.1)$$

By combining the likelihood function (3.1) and the joint prior function (3.2.5.1), the joint posterior distribution of u and v is given by

$$g_{34}\left(u,\frac{v}{X}\right) = \frac{\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)v^{-(\alpha_1+nu+1)}exp\{-v^{-1}(\beta_1+\sum_{i=1}^{n} x_i\}}{(\Gamma(u))^{n+1}}}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)\int_o^{\infty} v^{-(\alpha_1+nu+1)}exp\{-v^{-1}(\beta_1+\sum_{i=1}^{n} x_i)\}dv}{(\Gamma(u))^{n+1}}\right)}$$

$$= \frac{\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} ln x_i\right)v^{-(\alpha_1+nu+1)} exp\{-v^{-1}(\beta_1+\sum_{i=1}^{n} x_i\}}{(\Gamma(u))^{n+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n+1}(\beta_1+\sum_{i-1}^{n} x_i)^{\alpha_1+nu}}\right)}$$

The marginal posterior distribution of u and v are given below:

$$g_{34}\left(\frac{u}{X}\right) = \frac{\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\int_0^\infty v^{-(\alpha_1+nu+1)} exp\left\{-v^{-1}\left(\beta_1+\sum_{i=1}^{n} x_i\right)\right\}dv}{(\Gamma(\omega))^n}}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}$$

$$g_{34}\left(\frac{u}{X}\right) = = \frac{\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}$$

and,

$$g_{34}\left(\frac{v}{X}\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)}{(\Gamma(\omega))^{n+1}}\left(\frac{exp\left\{-v^{-1}\left(\beta_1+\sum_{i=1}^{n} x_i\right)\right\}}{v^{\alpha_1+nu+1}}\right)\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)} ; v > 0$$

Under squared loss function, the expression for Bayes estimators of u and v with their respective Posterior variance are given below:

$$\hat{u}_{34}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{u\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}$$

and,

$$\hat{v}_{34}/_X = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)}{(\Gamma(\omega))^{n+1}}\right)\left(\int_0^\infty \frac{v.exp\left\{-v^{-1}\left(\beta_1+\sum_{i=1}^{n} x_i\right)\right\}}{v^{\alpha_1+nu+1}}dv\right)}{\sum_{u=1}^{\infty}\left(\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)\right)}$$

$$= \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu-1)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu-1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}$$

Also,

$$Var\left(\hat{u}_{34}/_X\right) = \left(\frac{\sum_{u=1}^{\infty}\left(\frac{u\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(\omega))^{n+1}\left(\beta_1+\sum_{i-1}^{n} x_i\right)^{\alpha_1+nu}}\right)}\right) - \left(\hat{u}_{34}/_X\right)^2$$

and,

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$$Var\left(\hat{v}_{34}/_X\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} ln\, x_i\right)}{(\Gamma(u))^{n+1}}\right)\left(\int_0^{\infty}\frac{v^2.exp\{-v^{-1}(\beta_1+\sum_{i=1}^{n} x_i\}}{v^{\alpha_1+nu+1}}dv\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} ln x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n+1}(\beta_1+\sum_{i-1}^{n} x_i)^{\alpha_1+nu}}\right)} - \left(\hat{v}_{34}/_X\right)^2$$

$$Var\left(\hat{v}_{34}/_X\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u\, exp\left(u\sum_{i=1}^{n} ln\, x_i\right)\Gamma(\alpha_1+nu-2)}{(\Gamma(u))^{n+1}(\beta_1+\sum_{i-1}^{n} x_i)^{\alpha_1+nu-2}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u\, exp\left(u\sum_{i=1}^{n} ln\, x_i\right)\Gamma(\alpha_1+nu)}{(\Gamma(u))^{n+1}(\beta_1+\sum_{i-1}^{n} x_i)^{\alpha_1+nu}}\right)} - \left(\hat{v}_{34}/_X\right)^2$$

3.2.6 Posterior Distribution under Size Biased Poisson and Gamma priors

Again the prior for u is assumed to be Size BiasedPoisson distribution and for v the assumed prior is gamma distribution, having the following pdf

$$g_3(u,\theta_3) = \frac{e^{-\theta_3}\theta_3^{u-1}}{\Gamma(u)}\ ;\ \theta_3 > 0, u = 1,2,3, \ldots$$

$$g_5(v;\alpha_2,\beta_2) = \frac{\beta_2^{\alpha_2} v^{\alpha_2-1} exp(-v\beta_2)}{\Gamma(\alpha_2)}\ ; v > 0, \alpha_2 > 0, \beta_2 > 0$$

The joint Prior Distribution of u and v is defined as:

$$g_{35}(u,v) \propto g_3(u,\theta_3).\, g_5(v;\alpha_2,\beta_2) \qquad (3.2.6.1)$$

Combining the likelihood function (3.1) and the joint density (3.2.6.1), the joint posterior distributions of u and v are given by

$$g_{35}\left(u,{}^{v}/_X\right) = \frac{\dfrac{\theta_3^u v^{(\alpha_2-nu)-1} exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^{n} x_i)}{v}\right)\right\}}{exp\left(-u\sum_{i=1}^{n} ln\, x_i\right)(\Gamma(u))^{n+1}}}{\sum_{u=1}^{\infty}\left(\dfrac{\theta_3^u\int_o^{\infty} v^{(\alpha_2-nu)-1} exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^{n} x_i)}{v}\right)\right\}dv}{exp\left(-u\sum_{i=1}^{n} ln\, x_i\right)(\Gamma(u))^{n}}\right)}$$

$$g_{35}\left(u,{}^{v}/_X\right) = \frac{\dfrac{\theta_3^u v^{(\alpha_2-nu)-1} exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^{n} x_i)}{v}\right)\right\}}{exp\left(-u\sum_{i=1}^{n} ln\, x_i\right)(\Gamma(u))^{n+1}}}{\sum_{u=1}^{\infty}\left(\dfrac{2\theta_3^u exp\left(-u\sum_{i=1}^{n} ln\, x_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

The marginal posterior distribution of u and v are:

$$g_{35}\left({}^{u}/_{X}\right)=\frac{\dfrac{\theta_3^u\int_0^\infty v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv}{exp\left(-u\sum_{i=1}^n lnx_i\right)(\Gamma(u))^{n+1}}}{\sum_{u=1}^\infty\left(\dfrac{2\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$g_{35}\left({}^{u}/_{X}\right)=\frac{\dfrac{\theta_3^u exp\left(u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}((\sum x_i)^{-1}\beta_2)^{\alpha_2-nu}}}{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$g_{35}\left({}^{v}/_{X}\right)=\frac{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}}{exp\left(-u\sum_{i=1}^n lnx_i\right)(\Gamma(u))^{n+1}}\right)}{\sum_{u=1}^\infty\left(\dfrac{2\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

The expression for Bayes estimators of u and v with their respective Posterior variance are given below:

$$\hat{u}_{35}/_{X}=\frac{\sum_{u=1}^\infty\left(\dfrac{u\theta_3^u exp\left(u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}((\sum x_i)^{-1}\beta_2)^{\alpha_2-nu}}\right)}{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

and,

$$\hat{v}_{35}/_{X}=\frac{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u\int_0^\infty v.v^{(\alpha_2-nu)-1}exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^n x_i)}{v}\right)\right\}dv}{exp\left(-u\sum_{i=1}^n lnx_i\right)(\Gamma(u))^{n+1}}\right)}{\sum_{u=1}^\infty\left(\dfrac{2\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

$$\hat{v}_{35}/_{X}=\frac{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u exp\left(u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu+1}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu+1}}\right)}{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}$$

Also,

$$Var\left({}^{\hat{u}_{35}}/_{X}\right)=\frac{\sum_{u=1}^\infty\left(\dfrac{u\theta_3^u exp\left(u\sum_{i=1}^n lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}((\sum x_i)^{-1}\beta_2)^{\alpha_2-nu}}\right)}{\sum_{u=1}^\infty\left(\dfrac{\theta_3^u exp\left(-u\sum_{i=1}^n lnx_i\right)K_{K_{\alpha_2-nu}}\left(2\sqrt{\beta_2(\sum x_i)}\right)}{(\Gamma(u))^{n+1}\left(\sqrt{(\sum x_i)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)}-\left({}^{\hat{u}_{35}}/_{X}\right)^2$$

and,

$$Var\left({}^{\hat{v}_{35}}/_{X}\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u \int_0^{\infty} v.v^{(\alpha_2-nu)-1} exp\left\{\frac{-1}{2}\left((2\beta_2)v+\frac{(2\sum_{i=1}^{n}x_i)}{v}\right)\right\}dv}{exp\left(-u\sum_{i=1}^{n} lnx_i\right)\left(\Gamma(\omega)\right)^{n+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{2\theta_3^u exp\left(-u\sum_{i=1}^{n} lnx_i\right)K_{\alpha_2-nu}\left(2\sqrt{\beta_2\left(\sum x_i\right)}\right)}{\left(\Gamma(\omega)\right)^{n+1}\left(\sqrt{\left(\sum x_i\right)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)} - \left({}^{\hat{v}_{35}}/_{X}\right)^2$$

$$Var\left({}^{\hat{v}_{35}}/_{X}\right) = \frac{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(u\sum_{i=1}^{n} lnx_i\right)K_{\alpha_2-nu+1}\left(2\sqrt{\beta_2\left(\sum x_i\right)}\right)}{\left(\Gamma(\omega)\right)^{n}\left(\sqrt{\left(\sum x_i\right)^{-1}\beta_2}\right)^{\alpha_2-nu+1}}\right)}{\sum_{u=1}^{\infty}\left(\frac{\theta_3^u exp\left(-u\sum_{i=1}^{n} lnx_i\right)K_{K_{\alpha_2-nu}}\left(2\sqrt{\beta_2\left(\sum x_i\right)}\right)}{\left(\Gamma(\omega)\right)^{n+1}\left(\sqrt{\left(\sum x_i\right)^{-1}\beta_2}\right)^{\alpha_2-nu}}\right)} - \left({}^{\hat{v}_{35}}/_{X}\right)^2$$

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