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Skills in Mathematics Coordinate Geometry- Comparative Study of PSEB Students

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ABSTRACT

In the present research paper comparing the geometrical reasoning of primary and secondary school students was mainly based on the way students confronted and solved specific geometrical tasks: the strategies they used and the common errors appearing in their solutions. This comparison shed light to students' difficulties and phenomena related to the transition from Natural Geometry (the objects of this paradigm of geometry are material objects) to Natural Axiomatic Geometry (definitions and axioms are necessary to create the objects in this paradigm of geometry) and to the inconsistency of the didactical contract implied in primary and secondary school education. These findings stress the need for helping students progressively move from the geometry of observation to the geometry of deduction.

I.INTRODUCTION

Teaching geometry so that students learn it meaningfully requires an understanding of how students construct their knowledge of various geometric topics (Battista, 1999). This means it is necessary that mathematics educators investigate and mathematics teachers understand how students construct geometrical knowledge as a result of their learning experiences in school. An important aspect of this research direction is the study of the strategies that students use in different geometrical tasks as well as the identification of their mistakes. In the work of Piaget and in the Geneva School we see that errors were for the first time viewed positively, in the sense that they allow the tracing of the reasoning mechanisms adopted by students.

The literature review reveals that the investigation of various issues related to students' geometrical reasoning (knowledge, abilities, strategies, difficulties) is in most cases restricted to the study of groups that come from one educational level. We believe that it is necessary to gather empirical data which would allow the comparison between groups of students in primary and secondary education and would be valuable sources of information regarding aspects of teaching in the two educational levels as well as the difficulties met by students of different age groups.

The transition from elementary to secondary school is recognized as a critical life event, since, progressing from one level of education to the next, students may experience major changes in school climate, educational practices, and social structures (Rice, 2001). Research results reveal substantial agreement that there is often a decline in students' achievement following this transition, but achievement scores tend to recover in the year following the transition (Alspaugh, 1998). In the case of Punjab, students experience difficulty during the transition from elementary to secondary school which is evident in their performance in most topics, especially in mathematics.

This paper is based upon a research project which investigated the transition from elementary to secondary school geometry in Punjab, gathering data concerning students' performance in tasks involving two-dimensional geometrical figures, three-dimensional geometrical figures and net -representations of geometrical solids, as well as the students' spatial abilities. In the present paper we focus on the strategies the students used to solve specific geometrical tasks involving two-dimensional figures and we study the kinds of errors that we identified in the students' solutions.

II.THEORETICAL BACKGROUND

In the present paper we use as explanatory framework Duval's cognitive approach to geometry (Duval, 1995, 1998) and the framework of Geometrical Paradigms proposed by Houdement and Kuzniak (Houdement & Kuzniak, 2003; Houdement, 2007). We also use the concept of the didactical contract, introduced by Brousseau (1984) to interpret some of the students' wrong answers. According to him, the didactical contract is defined as a system of reciprocal expectancies between teacher and pupils, concerning mathematical knowledge. The didactical contract is in large part implicit and is established by the teacher in her teaching practice. The students may interpret the situation put before them and the questions asked to them on the basis of the didactical contract and act accordingly.

III.THE PRESENT STUDY

As noted in the introduction, this paper is based upon a research project which examined primary and secondary school students' geometrical knowledge and abilities related to tasks involving different geometrical figures, as well as their spatial abilities in micro-space. Participants in our study were 1000 primary and secondary school students. In the present paper we attempt to compare the geometrical reasoning of primary and secondary school students (the three age groups in our study) based on their solutions to three specific geometrical tasks which involved two-dimensional figures (the three tasks are shown in the Appendix). At this point we have to stress that the comparison attempted here does not refer to the levels of success of the three groups of students, since we study students of different age, from different educational levels, with different learning experiences and different cognitive abilities. Using as explanatory framework the theoretical notions presented above, we focus on the strategies and the common errors we identified in students' solutions. In this direction first we present part of the results from our study concerning students' solutions of three geometrical items included in the test and then we discuss these results and students' difficulties under the light of didactic phenomena rising from our research.

IV.RESULTS ON SPECIFIC GEOMETRICAL

ITEMS Item [A]

The results presented in Table 1 showed that while a high percentage of the students answered correctly to the specific multiple choice item (61.7% of 4th graders, 85.9% of 6th graders and 86.9% of 8th graders) – indicating they *know* that the four sides of a square are equal – a smaller number of students (especially from primary school) eventually gave a correct answer to the geometrical item [A].

Item	Answer	4th graders	6th graders	8th graders
Multiple choice	Correct	61.7	85.9	86.9
Item [A]	Correct – using properties	36.4	71.8	66.9
	Correct – applying theorem	---	---	18.5
	Wrong – using ruler	8.4	2.1	---
	Wrong – arithmetical operations	6.0	4.8	2.4

Table 1: Students' answers to multiple choice item and item [A] by age group

On the other hand, examining at the common errors identified in the students' solutions (Table 1), we notice some primary school students who gave (wrong) answers after using their ruler to measure the unknown segment on the geometrical figure presented on their paper. Additionally, a small number of students of the three age groups tried to combine the arithmetical data of the problem in a random way in arithmetical operations in order to come to an answer.

At this point it is interesting to state that, while the students could give the correct answer to item [A] by simply applying the property of equal sides in a square, we identified 18.5% of the secondary school students who solved the specific geometrical problem by applying Pythagoras' theorem in the subfigure of the right triangle. This performance is probably influenced by a part of the didactical contract according to which they are expected to apply Pythagoras' theorem any time a right triangle is involved in a geometrical figure. On the other hand, the specific performance indicates a difficulty concerning the transition from primary to secondary school. Specifically, the emphasis put on the use of algorithms during mathematics teaching in the secondary school seems to gradually result to the phenomenon that the students feel the safe of using an algorithm to be greater than that of a simple application of a geometrical property.

Items [B] and [C]

In Table 2 we present the results of students' attempts to solve two other geometrical tasks included in our test (item B and item C). Item [B] is a problem given to French students entering middle school (Duval, 2006). Item [C] was constructed for the present study, as an analogous problem to item [B], with two basic differences. First, on the geometrical figure presented in item [B], the subfigures of a circle and a rectangle appear, while on the geometrical figure presented in item [C] the two subfigures identified are a square and a rectangle. Second, the 'visibility' of the geometrical figure (and its subfigures) is less in the case of item [B] due to the specific configuration

Facing the geometrical problem presented in item [B] a number of students in the present study relied only on a visual perception of the figure (perceptual apprehension) and either considered point E as the middle of [AB] (16.5% of 6th grade students and 9.3% of 8th grade students), or answered that the length of segment [EB] is equal to the circle's ray, "because *it seems to be* equal to the ray" (11.1% of 6th grade students and 9.0% of 8th grade students).

	Item [B]			**Item [C]**		
Answer	4th graders	6th graders	8th graders	4th graders	6th graders	8th graders

Correct – using properties	15.1	33.3	51.9	46.1	62.2	81.5
Wrong – visual perception (i)	6.6	16.5	9.3	3.3	4.5	0.6
Wrong - visual perception (ii)	8.7	11.1	9.0			
Wrong – using algorithms	10.2	5.4	0.9	11.4	9.9	2.1

Table 2: Students' answers to item [B] and item [C] by age group

In order to solve the item [C], the solver had to identify the two subfigures, to possess and to use the cognitive unit referring to the property of equal sides of a square. As in the case of item [B], a number of students relied only on the visual perception of the given figure and considering point E as the middle of [AB] answered that the length of segment [EB] is equal to 3.5 cm. In both cases perceived features of the geometric figures (relying on a perceptual apprehension of the given figure in each problem) have misled the students as to the mathematical properties involved in the problem solution and have obstructed appreciation of the need for discursive apprehension of the presented geometrical figure.

Finally, it is interesting to note that, as in the case of item [A], there are (mainly primary school) students who tried to give an answer to the items [B] and [C] combining in arithmetical operations the data presented in the geometrical problems. A possible explanation to the specific students' performance is that, according to the implicit didactical contract (Brousseau, 1984) established during the teaching and learning processes in the mathematics classroom – especially the aspect concerning the solution of routine arithmetical word problems – when those students are given a geometrical problem which involves arithmetical data, they suppose that they are expected to combine them in order to give an answer. They probably consider that in this way not only they can give an answer, but they also demonstrate that they have tried to solve the problem by identifying and using the data given in the problem. So, they assume that their teacher will be pleased with their performance.

V.TEACHING IMPLICATIONS AND FURTHER RESEARCH

Most of the difficulties that have been identified and discussed in the present study concerning primary and secondary school students' attempts to solve geometrical problems are centred around the issue of the difficulties raised during the transition from Natural Geometry paradigm (where the objects are real, material) to

Natural Axiomatic Geometry paradigm (where the objects are conceptual). Subsequently, one of the main goals during the teaching of geometry should be to help students progressively pass from geometry where objects and their properties are controlled by perception to geometry where they are controlled by explicitation of properties. But, as Houdement and Kuzniak (2003) note, students and their teachers are not necessarily situated in the same geometrical paradigm, so this is a source of educational misunderstanding. Therefore, we consider essentially important that (prospective) primary and secondary school mathematics teachers are aware of the existence of the different geometrical paradigms (Houdement, 2007) and of the difficulties arising from the fact that plane geometrical figures on paper may be considered by the students in the teaching process during elementary school as if they were real objects (Berthelot & Salin, 1998). Further research is needed in order to prescribe and compare the way mathematics teachers in primary and secondary school approach geometry in their classrooms.

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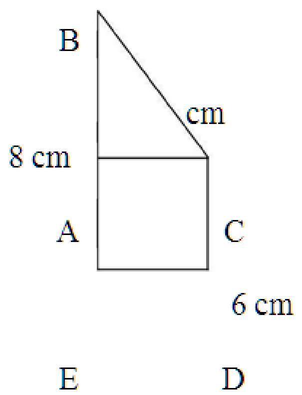
APPENDIX

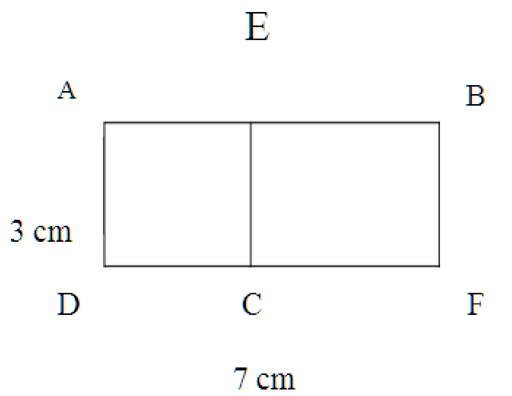
Item A

On the right triangle ABC, BC=10cm and AB=8cm. ACDE is a square (CD=6cm). Find the length of segment AC.



Item C

On the rectangle ABCD, DC=7cm and AD=3 cm. AEFD is a square. Find the length of segment EB.



Item B

On the figure sketched freehand here (the real lengths are written in cm), are represented a rectangle ABCD and a circle with center A, passing through D.

Find the length of segment EB.