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High Contact Ratio Spur Gear Mesh Stiffness and Load Sharing Ratio using Matlab & Excel Spread sheet

Vijay Karma[1], Ashish Kumar Agarwal[2]

[1]*Department of Mechanical Engineering, IET, Devi Ahilya Vishwavidyalaya, Indore, (India)*

[2]*Department of Mechanical Engineering, Vedant Institute of Management & Technology, (India)*

ABSTRACT:

The chief objective of this paper is to present an analytical approach for calculating the gearmeshstiffnesand lord sharing ratio using computer simulation with Matlab program. In this study, the effect of fillet-foundation deformation is also considered for calculating the gearmesh stiffness. Firstly, the behavior of pinion tooth, gear tooth and single tooth mesh stiffnesses are observed. Secondly, the total gearmesh stiffness behavior is analyzed along the pinion-roll angle variation. The effect of contact ratio is also investigated on gearmesh stiffness behavior using two models. Model I is considered for High contact ratio (HCR) spur gear pairs and model II for low contact ratio (LCR). The all the parameters are same for both the models except the addendum coefficient. The addendum coefficient is 1 for LCR gear pairs and 1.5 for the HCR gear pairs. The mean stiffnesses are also tabulated which describes the gearmesh stiffness behavior around along the pinion roll angle.. The effect of modules and pressure angles are also shown for twelve cases. And the results are validated by comparing the HCR gear pairs results with LCR gear pairs results. The results show that there is higher totalspur gearmesh stiffness for HCR gear pairs compared with LCR gear pairs. The whole analysis is performed with gear ratio 2.

Key Words – Gearmesh stiffness, module, pressure angle, high contact ratio, low contact ratio, load, sharing ratio.

1. INTRODUCTION

Gearing is one of the most operative approach of transferring rotary motion and torque from one shaft to another, with or without alteration of speed or direction of motion. A standard geared rotor system consists of a motor connected to one of the shafts by a coupling, a load at the other end of the other shaft and a gear pair which couples the shafts. Both shafts are supported at several locations by bearings. The coupling between torsional vibration modes is regulated by mesh stiffness. Spur gear mesh stiffness is assumed to be a time varying parameter. But, it is also the function of tooth geometrythat varies with the variation of number of teeth in contact,gear-pinion contact point, gear tooth deflections, gear tooth profile errors, gear hub torsional deflection and finally the local-faults on the tooth. The computation of gear mesh stiffness has become the most

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significant research area in gear dynamics. Various mathematical models have been developed to compute the mesh stiffness. These models can be classified mainly into three groups – experimental method, finite element method, analytical method. Among the three models, FE method and analytical method have extensively been used for mesh stiffness computation for the reason that it has great certainty.

2. SPUR GEARMESH STIFFNESS

In the present study, single tooth contact pairs, double tooth contact pairs and triple tooth contact pairs of gear system are investigated. In single pair, two teeth are meshed and share equal force. In case of double pair, two pairs share the total force and four teeth are meshed simultaneously. In case of triple pair, three pairs share the total force and six teeth are meshed simultaneously. In case of single tooth contact pair, the total effective mesh stiffness consists of gear 1 and gear 2 tooth mesh stiffness.The instantaneous single tooth contact pair mesh stiffness can be calculated as

$$K_{e(sp)} = \frac{1}{\frac{1}{K_1}+\frac{1}{K_h}+\frac{1}{K_2}}$$

(1)

Where, $K_1$and K_2 are pinion and gear tooth stiffnesses respectively and K_h is the Hertzian contact stiffness.

K_1andK_2 are the series combination of bending stiffness (K_b), axial compression stiffness (K_a), shear stiffness (K_s) and the stiffness due to fillet-foundation deflection (K_f) and can be calculated as:

$$\frac{1}{K_1} = \frac{1}{K_{b1}} + \frac{1}{K_{s1}} + \frac{1}{K_{a1}} + \frac{1}{K_{f1}}$$

(2)

And,

$$\frac{1}{K_2} = \frac{1}{K_{b2}} + \frac{1}{K_{s2}} + \frac{1}{K_{a2}} + \frac{1}{K_{f2}}$$

(3)

For single tooth meshing, the mesh stiffness is calculated as

$$K_{12} = \sum \frac{1}{\frac{1}{K_h}+\frac{1}{K_{a1}}+\frac{1}{K_{b1}}+\frac{1}{K_{s1}}+\frac{1}{K_{a2}}+\frac{1}{K_{b2}}+\frac{1}{K_{s2}}+\frac{1}{K_{f1}}+\frac{1}{K_{f2}}}$$

(4)

For multi pairs, the total effective mesh stiffness is parallel combination of instantaneous single pairs mesh stiffness in the direction of force that can be written as:

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$$K(t) = \sum_{i=1}^{n} \frac{1}{\frac{1}{K_h}+\frac{1}{K_{a1}}+\frac{1}{K_{b1}}+\frac{1}{K_{s1}}+\frac{1}{K_{a2}}+\frac{1}{K_{b2}}+\frac{1}{K_{s2}}+\frac{1}{K_{f1}}+\frac{1}{K_{f2}}}$$

(5)

The Bending, Shear, Axial compression, Hertzian Contact and Fillet-foundation deflections of the gear tooth are calculated based on the model developed by reference [1-7].

3. LOAD SHARING RATIO

In high contact ratio spur gear pairs, there are maximum three teeth in contact. For high contact ratio spur gear pairs, the load shared at contact points are calculated as:

$$F_1 = F\frac{K_{e1}}{K_{e1}+K_{e2}+K_{e3}}$$

(6)

$$F_2 = F\frac{K_{e2}}{K_{e1}+K_{e2}+K_{e3}}$$

(7)

$$F_3 = F\frac{K_{e3}}{K_{e1}+K_{e2}+K_{e3}}$$

(8)

In the similar way, there are two teeth in contract for low contact ratio spur gear pair.in such a case, the load shared is calculated as:

$$F_1 = F\frac{K_{e1}}{K_{e1}+K_{e2}} \qquad (9)$$

$$F_2 = F\frac{K_{e2}}{K_{e1}+K_{e2}} \qquad (10$$

4. CONTACT RATIO

The contact ratio [8] for a pair of two external spur gears is

$$\varepsilon = \frac{z_1}{2\pi} * (\tan\alpha_{a1} - \tan\alpha) + \frac{z_2}{2\pi} * (\tan\alpha_{a2} - \tan\alpha) \qquad (11)$$

Where, $\cos\alpha_{a1} = \frac{z_1 * \cos\alpha}{2R_{a1}}$ and $\cos\alpha_{a2} = \frac{z_2 * \cos\alpha}{2R_{a2}}$ (12)

An analytical approach is used to solve the used model for computing high contact ratio spur gear mesh stiffness which is explained earlier [9,10]. This approach can also be used for normal conditions by using addendum coefficient as 1.Two models of spur gear pairs are analyzed with different modules and different pressure angle

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combinations. The main gear parameters of the spur gear transmission system are given in Table 1. The complete detail of all cases is shown in Table 2.

Table 1. Parameters of the gear-pinion set

Parameter	Model I (LCR) Pinion/Gear	Model II (HCR) Pinion/Gear
Tooth shape	Standard involute	Standard involute
Material	Steel	Steel
Number of Teeth z	30/60	30/60
Young's modulus E (GPa)	210	210
Poisson ratio υ	0.3	0.3
Module m (mm)	3,4,5,6	3,4,5,6
Pressure angle α (^{0})	18,20,22	18,20,22
Tip clearance coefficient c^*	0.25	0.25
Addendum coefficient h_a^*	1	1.5
Face width L (mm)	10m	10m
Hub bore radius r_{int}(mm)	17.5	17.5
Contact Ratio ε	<2	>2

Table 2. Cases to be studied

Case	Module (mm)	Pressure Angle (Degree)	Contact Ratio (LCR)	Contact Ratio (HCR)
1	3	18	1.8277	2.5884
2	3	20	1.7191	2.4528
3	3	22	1.6269	2.3360
4	4	18	1.8277	2.5884
5	4	20	1.7191	2.4528
6	4	22	1.6269	2.3360
7	5	18	1.8277	2.5884
8	5	20	1.7191	2.4528
9	5	22	1.6269	2.3360
10	6	18	1.8277	2.5884
11	6	20	1.7191	2.4528
12	6	22	1.6269	2.3360

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5. NUMERICAL RESULTS

A fluctuation of 1 to 2teeth pairs in contact is followed for low contact ratio (ε <2) gear pairs. And, fluctuation of 2 to 3 pairs in contact is followed for high contact ration (ε>2) gear pairs. This affects the gear mesh stiffness K(t). This variation is considered as the main excitation source of vibration and acoustic emissions. The total gear roll angle corresponding to a spur gear tooth traversing the load zone equals the contact ratio(ε) times the angle, a single gear tooth rotates before reaching the relative position with respect to a pinion teeth: $\theta_e = \frac{360}{z_1}$.

The sequence of number of tooth pairs in mesh is two-one-two for low contact ratio gear pairs. In this same respect, the sequence of number of tooth pairs is three-two-three for high contact ratio gear pairs. The roll angle varying gearmesh stiffness in calculated point by point by a summation of the single pair gearmesh stiffness so that the maximum values of stiffness corresponding to two pairs in contact are obtained from ($i\theta_e$) to ($\varepsilon - 1)((i + 1)\theta_e$) and the minimum values corresponding to one pair in contact are obtained from (ε − 1)((i+1)θ_e) to ((i + 1)θ_e) for low contact ratio gear pair withi integer. Similarly, maximum values of stiffness corresponding to three pairs in contact are obtained from ($i\theta_e$) to ($\varepsilon - 2) \times ((i + 1)\theta_e$) and the minimum values corresponding to two pair in contact are obtained from (ε − 2)×((i+1)θ_e) to ((i + 1)θ_e) for high contact ratio gear pair.

Two sets of spur gear-pinion pair are studied with various modules and pressure angle combinations. The main gear parameters of the studied transmission are given in Table 1. The complete detail of all cases is shown in Table 2.These have the same parameters except for addendum coefficient and contact ratio. Model I is a collection of spur gear sets with low contact ratio and Model II with high contact ratio. With these gear models, the impact of various modules and pressure angles on mesh stiffness can be carried out for all the cases with LCR and HCR. Their computation procedure has been explained in [9,10]. Pinion roll angle corresponding to case 1 (HCR pair) in computed as:

$$\theta_p = \varepsilon \frac{360}{z_1} = 31.0608 \text{ deg.}$$

For the pinion roll angle varying mesh stiffness calculation, firstly, the locations of the one and two pairs of teeth contact zones are determined from the contact ratio and number of teeth on pinion of the LCR gear pairs and two and three pairs of teeth contact zones for HCR gear pairs. The formulas of the stiffnessare given in Eqs. (1 - 5). Depending of the contact ratio, the various cases are determined and the mesh stiffness of gear pair is obtained according to Eqs. (4) and (5). For standard gear parameters given in Table 1, a matlab program is written and numerical values of mesh stiffness are obtained.

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5.1 SINGLE TOOTH MESH STIFFNESS OF SPUR GEAR PAIR

For case 1, the single tooth mesh stiffness within one mesh cycle is plotted in Fig. 3 and Fig. 4. Fig. 3 represents the evolution of gearmesh stiffness along the pinion roll angle corresponding to module 3 mm and pressure angle 18 deg. for HCR spur gear pair and Fig. 4 for LCR spur gear pair, respectively. The variation of pinion and gear tooth stiffness along the path of contact is also shown in same figure. It has been observed that stiffness of the driving gear tooth decreases, whereas the stiffness of the driven gear tooth increases along the path of contact. This is due to the fact that on the driving gear, the contact point moves from root to the tip of the tooth whereas on the driven gear, the contact point moves from tip to the root.

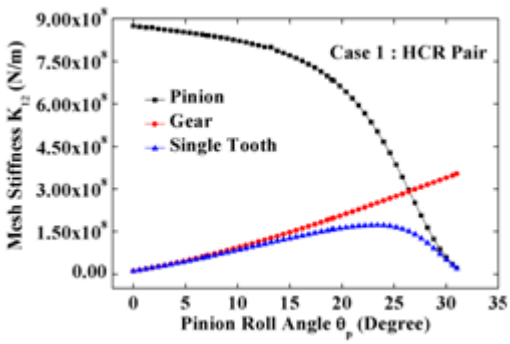

Fig. 3.

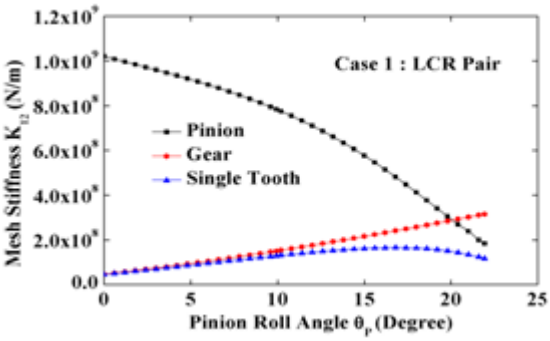

Fig. 4.

Fig. 3. Case 1 (HCR gear pair) : The Pinion tooth, Gear tooth and Single tooth mesh stiffness K_{12}, vs. the pinion roll Angle, θ_p, within one mesh cycle, when the gear teeth are perfect.

Fig. 4. Case 1 (LCR gear pair) : The Pinion tooth, Gear tooth and Single tooth mesh stiffness K_{12}, vs. the pinion roll Angle, θ_p, within one mesh cycle, when the gear teeth are perfect.

5.2 TOTAL MESH STIFFNESS OF SPUR GEAR PAIR

The obtained evolution of gearmesh stiffness is the typical one for the gear systems characterized by the maximum in zone of three tooth pairs in contact and minimum in zone of two tooth pairs in contact for HCR gear pairs. In the same way, gearmesh stiffness evolution is also typical on for LCR gear pairs with maximum in contact zone of two tooth pairs and minimum in contact zone of one tooth pair. The total gearmesh stiffness mean values are tabulated in Table 3 for both the models. In table 3, there is also shown the percentage difference in the total gearmesh stiffness for HCR gear pairs and LCR gear pairs. This percentage difference shows the amount to which the mean value of total gearmesh stiffness is higher for HCR gear pairs than for LCR gear pairs.Fig. 5 represents the evolution to total mesh stiffness along the pinion roll angle with one-third shaft rotation for High contact ratio spur gear pairs. Fig. 6 represents the evolution to total mesh stiffness along the pinion roll angle with one-third shaft rotation for Low contact ratio spur gear pairs.

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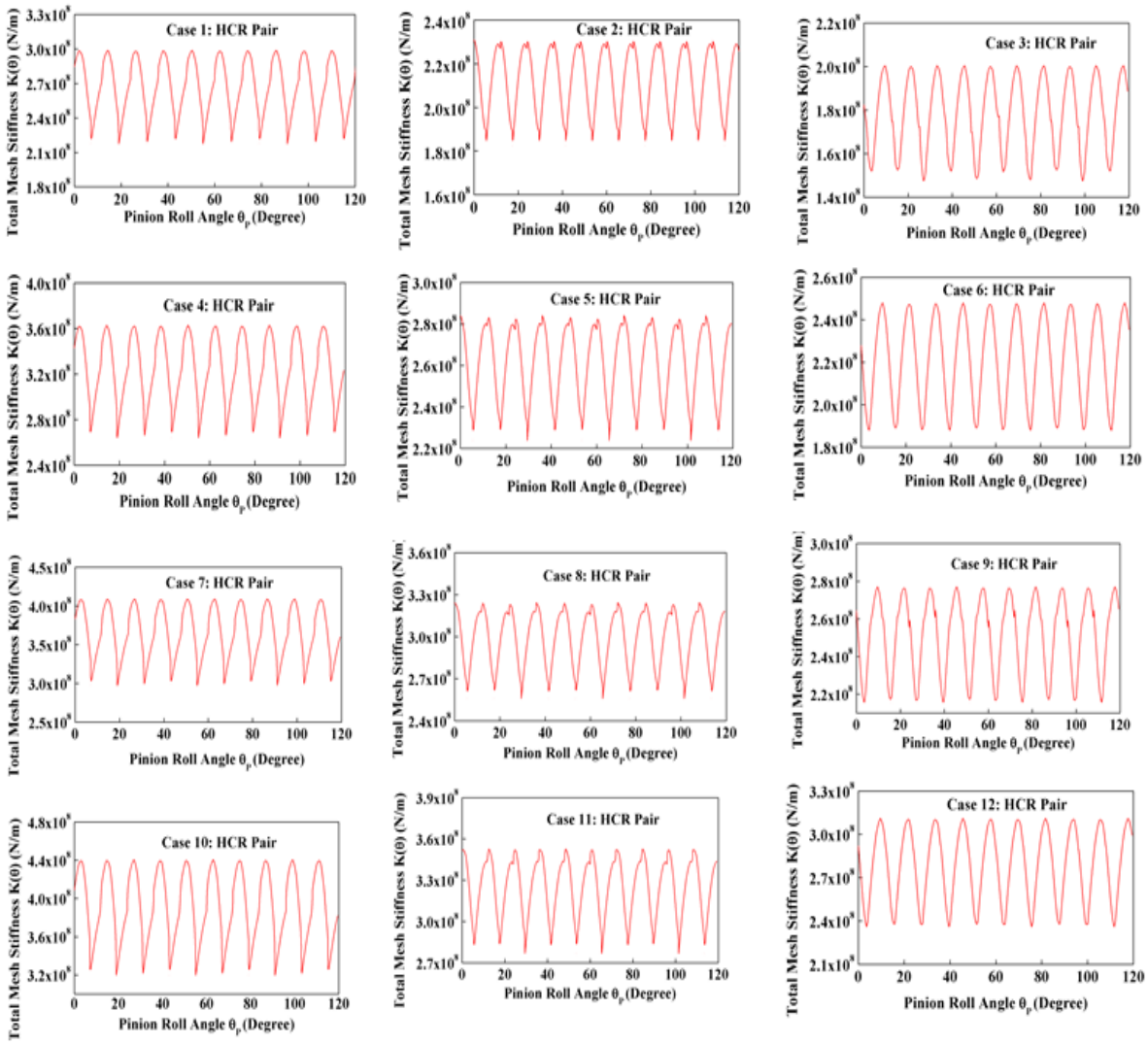

Fig. 5.HCR Spur gearTotal mesh stiffness K(θ), vs. the pinion roll Angle, θ_p, within one third shaft rotation, when the gear teeth are perfect.

5.3 EFFECT OF PRESSURE ANGLE ON TOTAL GEARMESH STIFFNESS

From the plots and table 3, it is seen that total mesh stiffness decreses with increasing pressure angle for a particular module and number of teeth on pinion and gear. This behaviour is same for both the models. For validation, the obtained results for the HCR gear pairs are compared with the same for the LCR gear pairs.it can be seen with the table 3 that there is higher gearmesh stiffness for HCR gear pairs than same for LCR gear pairs. But, with increasing the pressure angle, the difference between gearmesh stiffnesses decreases that reduces the usefulness of HCR gear pairs, when compared to LCR gear pair.

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Table 3. Total gearmesh stiffness (MN/m)

Case	Module (mm)	Pressure Angle (^{0})	HCR pair	LCR pair	Difference (%)
1	3	18	267.20	225.73	18.37
2	3	20	210.69	197.30	6.79
3	3	22	174.49	173.64	0.49
4	4	18	324.63	274.42	18.30
5	4	20	259.50	241.25	7.56
6	4	22	216.40	214.00	1.12
7	5	18	365.99	307.77	18.92
8	5	20	296.77	273.53	8.5
9	5	22	246.66	244.62	0.83
10	6	18	393.77	328.91	19.72
11	6	20	323.26	295.32	9.46
12	6	22	273.26	266.18	2.66

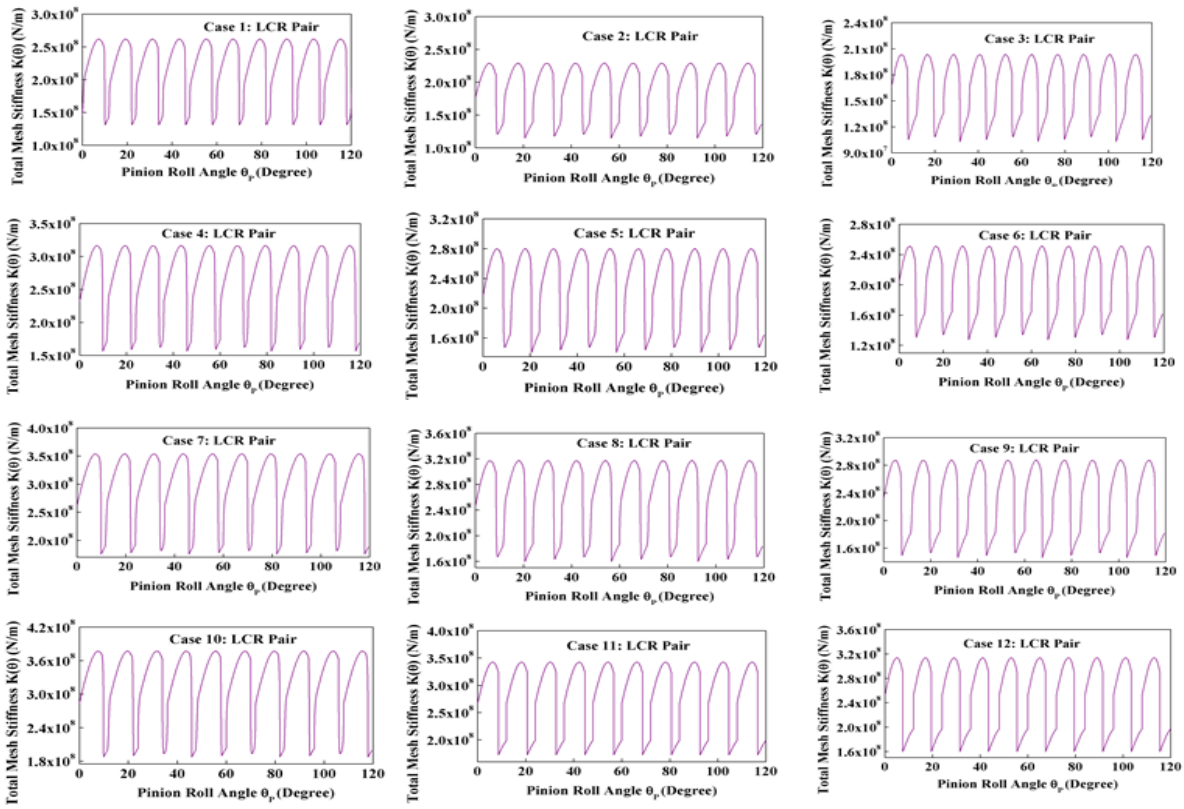

Fig. 6.LCR Spur gearTotal mesh stiffness K(θ), vs. the pinion roll Angle, θ_p, within one third shaft rotation

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5.4 VARIATION OF LOAD SHARING RATIO

The variations of individual tooth normal load (in percentage) along the path of contact for various pinion roll angles are shown in Fig. 7 and Fig. 8. These figures show that the normal load reaches its maximum in the two pairs contact zone for high contact ratio spur gear pairs.For the HCR spur gear pairs (as shown from Fig, 7), it can be seen that the percentage of normal load per tooth is high in the regions of two teeth pairs contact while load is low in the regions of three teeth pairs in contact for high contact ratio spur gears. But, total normal load is not occurred at any point along the path of contact in high contact ratio spur gear pairs.

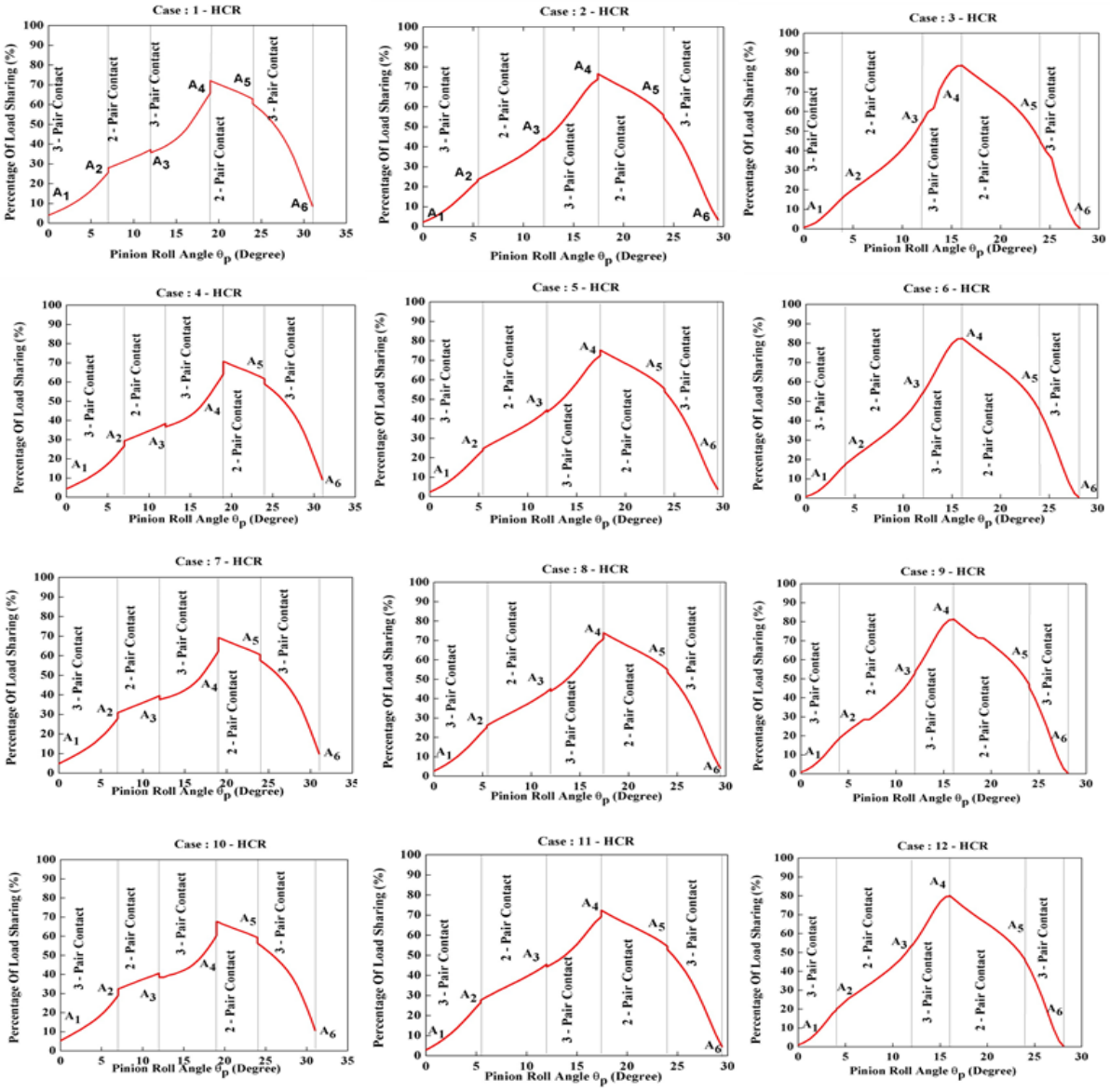

Fig. 7. HCR Spur Gear Pair Load shared in percentage Vs. the pinion rollAngle, $\boldsymbol{\theta p}$, for one mesh cycle

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In the same way, for the LCR spur gear pairs (as shown from Fig. 8), the percentage of normal load per tooth is high in the regions of one tooth pair contact while load is low in the regions of two teeth pairs in contact for low contact ratio spur gears. It is due to the load sharing between teeth pairs. In LCR spur gear pairs, total normal load is occurred at all points in single tooth pair zone along the path of contact.

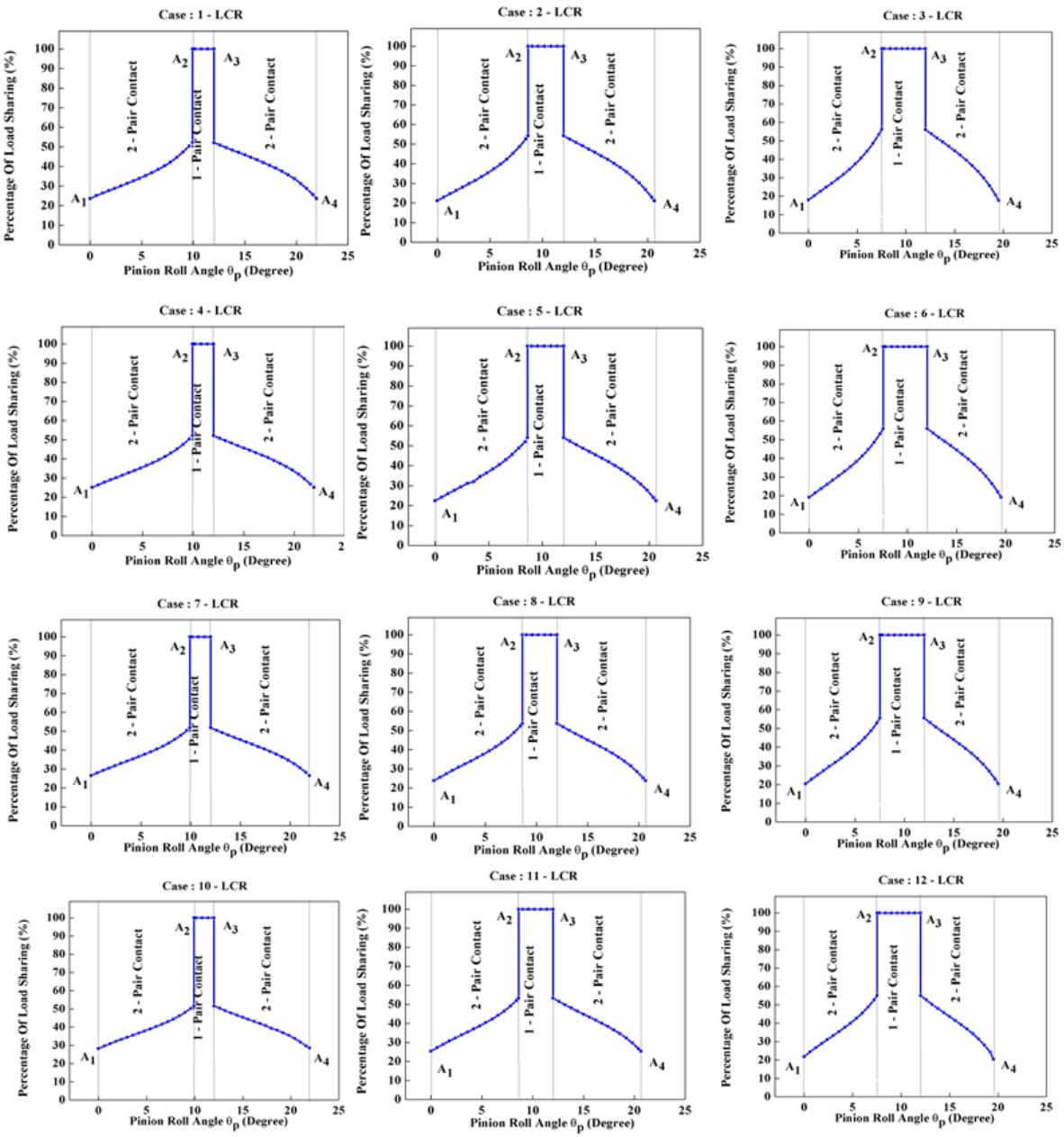

Fig. 8. LCR Spur Gear Pair Load shared in percentage Vs. the pinion rollAngle, $\boldsymbol{\theta p}$, for one mesh cycle

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In case of High contact ratio spur gear pairs, the percentage of load shared per tooth is highin the region where two pairs in contact and it decreases when it approaches to the region of three pairs in contact. In the same way, In case of Low contact ratio spur gear pairs, the percentage of load shared per tooth is highin the region where one pair in cointact and it decreases when it approaches to the region of two pairs in contact.

6. CONCLUSION

In this study, an analytical approach using computer simulation is applied for calculating pinion-roll angle varying mesh stiffness considering the effect of fillet-foundation deformation. A pair of meshing spur gears consisting of a perfect gear has been investigated. The numerical values are the initial parameters for visualising the gearmesh stiiffness behaviour along the pinion roll angle. It has been found that for a particular module, gear mesh stiffness decreases with increasing the pressure angle. But for a particular pressure angle, no much variation has been found with increasing the module. The model II results are considered as the refernece and the results for model I are compared with the model II. It is also found that there is higher gearmesh stiffness for HCR gear pairs compared with LCR gear pairs. For example, the total gearmesh stiffness is 18.37% higher for case 1 HCR gear pair than the same for LCR gear pair. In HCR spur gear pairs, load shared by single gear tooth does not reach maximum applied load. It is always less than the total applied load. In LCR spur gear pairs, load shared by single tooth reaches maximum applied load in single pair contact zone which is main cause of noise and vibration. Future research topics iclude the statistical parameters analysis using the gearmesh stiffness numerical values for better understanding the gearmesh stiffness variation along the pinion roll angle.

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