Box Counting Method for Fractal Dimension Analysis

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Abstract— **Fractal Dimensional analysis has a wide application in the field of surface analysis and study of structure of different materials. In this paper we present a review on fractal dimensional analysis of any image by iterated function system and partitioned iterated function system. Fractal dimensional calculation for a particular part of any image can be done effectively by Partitioned Iterated Function System. This study may be very useful for surface analysis of AFM images of different materials.**

Keywords—Fractal Dimension, Iterated Function System, Partitioned Iterated Function System.

I. INTRODUCTION

Fractal is an infinitely complex pattern of self similar images of any scale. The repetition of these images has never ended. Fractal geometry was first introduced by Mandelbrot for self-similar sets of images [1]. Fractal geometry has wide application in the field of image compression, estimation of fractal dimension, estimation of surface roughness, simple models of complex structures etc. [1-3]. For estimation of fractal dimension different methods such as Gangepain and Roques-carmes presented reticular cell counting method [4,5] and later on Keller et al. proposed probability box counting [4,6]. Both these methods are good but could not give satisfactory results in some cases. To avoid these conditions a modified method named Box counting method was established by Sarkar et. al [4,6,7,8]. This paper presented the essential ideas about the calculation of fractal dimension. The change in fractal dimension is based on image compression which has done by Iterated Function System. Iterated Function System was for image compression was first given by M.F. Baransley [9]. The concept of Partitioned Iterated Function System (PIFS) image compression through PIFS was done previously in [10, 11]. In this paper we present a review on fractal dimensional analysis of any image by iterated function system and partitioned iterated function system. Fractal dimensional calculation for a particular part of any image can be done effectively by PIFS. This study may be very useful for surface analysis of AFM images of different materials.

II. FRACTAL DIMENSION

Rough fragmental geometry that are self similar and irregular in nature are termed as fractal. Fractal geometry was first introduced by Mandelbrot in 1982. Many literature evidence are available that shows the broad application of fractal analysis for the study of image or object. The main feature which is used for analysis of image or object is measurement of fractal dimension.
The dimension is simply the exponent of the number of self-similar pieces with magnification factor **N** into which the figure may be broken.

A square can be break into 4,9,25… etc. self similar pieces with magnification factor 2,3 and 5…respectively. So, we have $\mathbf{N^2}$ self-similar pieces, each with magnification factor **N**. So we can write

$$dimension = \frac{\log\,(number\ of\ self-similar\ pieces)}{\log\,(magnification\ factor)}$$

$$= \frac{\log N^2}{\log N}$$

$$= \frac{2\log N}{\log N}$$

$$= 2$$

Similarly, cube is also break into N^3 self-similar pieces with scale N.

The dimension of a cube is

$$dimension = \frac{\log\,(number\ of\ self-similar\ pieces)}{\log\,(magnification\ factor)}$$

$$= \frac{\log N^3}{\log N}$$

$$= \frac{3\log N}{\log N}$$

$$= 3$$

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Thus, we take as the *definition* of the fractal dimension of a self-similar object

$$Fractal\ dimension = \frac{\log\ (number\ of\ self-similar\ pieces)}{\log\ (magnification\ factor)}$$

Review of literature shows that there are a number of ways to determine the fractal dimension of a given image or object. The different methods for calculating fractal dimension are walking-divider method, Box counting method, Prism counting method, Epsilon-Blanket method, Perimeter-Area Relationship, Fractional Brownian Motion, Power spectrum method and Hybrid method.

All the above methods box counting method and power spectrum method are very interesting and accurate.

III. Iterated Function System

Fractal dimension calculation is done by IFS by calculating attractor. The attractor is obtained by applying contractions.

Let . A mapping is called a contraction on X if there is a number s with such that

$|T(x) - T(y)| \leq s|x - y| \quad for\ all\ x, y \in X.$

Here s is a contractivity factor. The mapping T is a group of different mapping.

$T = \{T_1, T_2, \ldots .T_n\}, with\ n \geq 2.$

This mapping is known as iterated functions system. By applying these iterations on a unction we get a non-empty compact subset A of X. This A is known as an attractor.

IFS is analogue with Banach contraction principle and attractor as fixed point. Attractor A is defined as

$$A = \bigcup_{i=1}^{n} T_i(A)$$

The main property of an iterated function system is that it determines the unique attractor which is generally known as fractal.

If are similarities or iterations then

$|T_i(x) - T_i(y)| \leq s_i|x - y| \ \ where\ 0 < s_i < 1,$

is called ratio of . Each transforms subset of into geometrically similar sets. The attractor of such a collection is known as self similar set. It is also known as union of small self similar copies.

One of the advantages of using an Iterated Function System is that the dimension of the attractor is often relatively easy to estimate in terms of the defining contractions. A dimension contains much informacion about the geometrical properties of a set. The Hausdorff dimension and box dimension of the attractor of an Iterated Function System consisting of contractions are very useful to calculate fractal dimension.

A. Box Counting Method

Box counting method is the most popular method for computing the fractal dimension of one dimensional and two dimensional data. The box counting method is analogous to the perimeter measuring method we used for the coastlines. This method is originated by Voss in 1985. In this method, a grid of n- dimensional boxes or hyper-cubes are formed on the fractal surface and and then count how many boxes of the grid are covering part of the image. For fractal analysis of images, the fractal surface is covered with cubes. The fractal surface is covered with boxes of size where is a power of 2. Using the box counting method a graph is plotted between on the Y-axis and value of on the X-axis. The slope of the line shows the fractal dimension of the image. The fractal dimension of the fractal image is given by

$Fractal\ Dimension\ = -\beta$

Dimension calculated by box counting method is known as the box or Minkowski dimension. For a smooth one-dimensional curve, it is expected that

$N\ = Log\ N/Log\ \delta$

Where the length of the curve is, is the number of nonempty boxes and is the size of the box. According to data a regular arrangement of boxes are formed on the surface of fractal image and the non-empty boxes are counted. If the value of slope is 1 it means it is a straight line. If fractal dimension is more than 2 it shows the image is more fractally and the image is more complex. Sarkar and Chaudhuri (1992) have considered the problem in a two-dimensional version of this algorithm for optimizing the number of boxes for a given size required to compute a fractal dimension close to the Hausdorff dimension.

IV. Partial Iterated Function System

The basic IFS is not coded the real world images efficiently in some cases. For this reason coding is done by introducing the concept of partial iterated function system. PIFS codes are obtained and applied to a particular portion of any image in place of the whole image.

Let an image of size $m \times m$. This image is partitioned into n non-overlapping squares of size $a \times a$. The partitioned is represented by $L = \{L_1, L_2, L_3, \ldots \ldots L_n\}$. Here $n = \frac{m}{b} \times \frac{m}{b}$ and each L_i is known as Range-blocks.

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Let there is another collection of blocks having size $2a \times 2a$. This collection is given by $M = \{M_1, M_2, \ldots \ldots . M_p\}$. This M is known as Domain block with $p = (w - 2a) \times (w - 2a)$.

Let $R_j = \{f : f(M_i), f \text{ is an affine map}\}$

For a given range block L_i and mapping $f_{i|j} \in R_j$ be such that

$$d\left(L_i, f_{i|j}(M_j)\right) \leq d\left(L_i, f(M_j)\right) \text{ where } f \in F_j.$$

For every range block L_i, there must be a domain block M_k. The set of maps $F = \{f_1, f_2, \ldots . . f_n\}$ thus obtained is called the Partial Iterated Function System (PIFS).

A. Difference between IFS and PIFS

The main difference between PIFS and IFS is that: in PIFS the transformations are not applied to the whole image and applied only for specified domains. Other difference between PIFS and IFS is for contractivity factor of the affine transformation. PIFS have expansive transformation and a bounded attractor. In case of IFS and expansive map does not to converge at a common fixed point.

B. Fractal Dimension by PIFS

A fractal dimension by partitioned iterated function system is used to calculate the fractal dimension FD of a surface, according to the following formula:

$$FD = 2 + \frac{log(\sum_{i=1}^{N} |r_i| \, d_i)}{\log(2)}$$

Where N is the number of range, is the scale coefficient and is the domain factor.

The domain factor is given by where , is the area covered by the domain and denotes the sum of all areas covered by the domain. k is the inverse of the scale factor between range sizes and domain sizes. This particular fractal method has many advantages over other methods of calculating fractal. One of the important advantages is that it applies on a fractal surface in which scaling factors are defined locally for each affine transformation.

There is a good relation between fractal dimension and surface roughness. If the fractal dimension is high the surface is more complex.

V. CONCLUSION

In this paper we study about Fractal dimension and fractal dimensional analysis of any image by iterated function system and partitioned iterated function system. A comparative study between iterated function system and partitioned iterated function system is also included in this paper. This study may be very useful for surface analysis of AFM images of different materials.

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