



ON $(1,2)^*-(sp)^*$ -CLOSED SETS

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ABSTRACT

In this paper, we introduce a new class of sets called $(1,2)^*-(sp)^*$ -closed sets in bitopological spaces. We prove that this class lies between the class of $\tau_{1,2}$ -closed sets and the class of $(1,2)^*$ -g-closed sets. Also we find some basic properties of $(1,2)^*-(sp)^*$ -closed sets. Applying these sets, we introduce a new space called $T_{(1,2)^*(sp)^*}$ -space.

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1. INTRODUCTION

Ravi and Lellis Thivagar [6] introduced the concepts of $(1,2)^*$ -semi-open sets, $(1,2)^*$ - α -open sets, $(1,2)^*$ -generalized closed sets and $(1,2)^*$ - α -generalized closed sets in bitopological spaces. Jafari et al [2] introduced the notion of $(1,2)^*$ - $\hat{\alpha}g$ -closed sets and investigated its fundamental properties. In this chapter we introduce a new class of sets called $(1,2)^*-(sp)^*$ -closed sets in bitopological spaces and prove that this class lies between the class of $\tau_{1,2}$ -closed sets and the class of $(1,2)^*$ -g-closed sets. Further we introduce a new space called $T_{(1,2)^*(sp)^*}$ -space.

2. PRELIMINARIES

Throughout this paper, X and Y denote bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) , respectively, on which no separation axioms are assumed.



Definition 2.1 [6]

Let S be a subset of X . Then S is said to be $\tau_{1,2}$ -open if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.

We call $\tau_{1,2}$ -closed set is the complement of $\tau_{1,2}$ -open.

Example 2.2

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$ -open and the sets in $\{\phi, \{a\}, \{b\}, X\}$ are called $\tau_{1,2}$ -closed.

Definition 2.3 [6]

Let S be a subset of X . Then

- (i) The $\tau_1\tau_2$ -interior of S , denoted by $\tau_1\tau_2$ -int(S) or $\tau_{1,2}$ -int(S), is defined by $\cup \{F : F \subset S \text{ and } F \text{ is } \tau_{1,2}\text{-open}\}$.
- (ii) The $\tau_1\tau_2$ -closure of S , denoted by $\tau_1\tau_2$ -cl(S) or $\tau_{1,2}$ -cl(S), is defined by $\cap \{F : S \subset F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$.

Remark 2.4 [6]

Notice that $\tau_{1,2}$ -open sets need not necessarily form a topology.

Remark 2.5 [3, 6]

Without proving we list the following properties for the bitopological space (X, τ_1, τ_2) , where $\tau_{1,2}$ -open subsets are defined as above.

(P0) If $S_1 \subset S_2 \subset X$, then $\tau_1\tau_2$ -int(S_1) \subset $\tau_1\tau_2$ -int(S_2) and $\tau_1\tau_2$ -cl(S_1) \subset $\tau_1\tau_2$ -cl(S_2).

(P1) (a) $\tau_1\tau_2$ -int(S) is $\tau_{1,2}$ -open for each $S \subset X$;

(b) $\tau_1\tau_2$ -cl(S) is $\tau_{1,2}$ -closed for each $S \subset X$.

(P2) (a) A set $S \subset X$ is $\tau_{1,2}$ -open if and only if $S = \tau_1\tau_2$ -int(S);

(b) A set $S \subset X$ is $\tau_{1,2}$ -closed if and only if $S = \tau_1\tau_2$ -cl(S).

(P3) (a) For any $S \subset X$ we have $\tau_1\tau_2$ -int($\tau_1\tau_2$ -int(S)) = $\tau_1\tau_2$ -int(S);



- (b) For any $S \subset X$ we have $\tau_1 \tau_2 - \text{cl}(\tau_1 \tau_2 - \text{cl}(S)) = \tau_1 \tau_2 - \text{cl}(S)$.
- (P4) (a) $\tau_1 \tau_2 - \text{int}(X - S) = X - \tau_1 \tau_2 - \text{cl}(S)$ for any $S \subset X$;
- (b) $\tau_1 \tau_2 - \text{cl}(X - S) = X - \tau_1 \tau_2 - \text{int}(S)$ for any $S \subset X$.
- (P5) (a) $\tau_1 \tau_2 - \text{int}(S) = \text{int} \tau_1(S) \cup \text{int} \tau_2(S)$ for any $S \subset X$;
- (b) $\tau_1 \tau_2 - \text{cl}(S) = \text{cl} \tau_1(S) \cap \text{cl} \tau_2(S)$ for any $S \subset X$.
- (P6) For any family $\{S_i / i \in I\}$ of subsets of X we have :
- (a₁) $\bigcup_i \tau_1 \tau_2 - \text{int}(S_i) \subset \tau_1 \tau_2 - \text{int}(\bigcup_i S_i)$;
- (b₁) $\bigcup_i \tau_1 \tau_2 - \text{cl}(S_i) \subset \tau_1 \tau_2 - \text{cl}(\bigcup_i S_i)$;
- (a₂) $\tau_1 \tau_2 - \text{int}(\bigcap_i S_i) \subset \bigcap_i \tau_1 \tau_2 - \text{int}(S_i)$;
- (b₂) $\tau_1 \tau_2 - \text{cl}(\bigcap_i S_i) \subset \bigcap_i \tau_1 \tau_2 - \text{cl}(S_i)$.

We recall the following definitions which are useful in the sequel.

Definition 2.6

A subset A of a bitopological space (X, τ_1, τ_2) is called

- (1) $(1,2)^*$ -semi-open [10] if $A \subseteq \tau_{1,2} - \text{cl}(\tau_{1,2} - \text{int}(A))$.
- (2) $(1,2)^*$ - α -open [4, 10] if $A \subseteq \tau_{1,2} - \text{int}(\tau_{1,2} - \text{cl}(\tau_{1,2} - \text{int}(A)))$.
- (3) $(1,2)^*$ - β -open [11] if $A \subseteq \tau_{1,2} - \text{cl}(\tau_{1,2} - \text{int}(\tau_{1,2} - \text{cl}(A)))$.

The complement of a $(1, 2)^*$ -semi-open (resp. $(1,2)^*$ - α -open, $(1,2)^*$ - β -open) set is called $(1,2)^*$ -semi-closed (resp. $(1,2)^*$ - α -closed, $(1,2)^*$ - β -closed).

The $(1,2)^*$ - α -closure [2, 7] (resp. $(1,2)^*$ -semi-closure [2, 7], $(1,2)^*$ - β -closure [16, 17]) of a subset A of X , denoted by $(1,2)^*$ - α cl(A) (resp. $(1,2)^*$ -scl(A), $(1,2)^*$ - β cl(A)) is defined to be the intersection of all $(1,2)^*$ - α -closed (resp. $(1,2)^*$ -semi-closed, $(1,2)^*$ - β -closed) sets of X containing A . It is known that $(1,2)^*$ - α cl(A) (resp. $(1,2)^*$ -scl(A), $(1,2)^*$ - β cl(A)) is a $(1,2)^*$ - α -closed (resp. $(1,2)^*$ -semi-closed, $(1,2)^*$ - β -closed) sets. For



any subset A of an arbitrarily chosen bitopological space, the $(1,2)^*$ - α -interior [2, 7] (resp. $(1,2)^*$ -semi-interior [2, 7], $(1,2)^*$ - β -interior [16, 17]) of A , denoted by $(1,2)^*$ - α int(A) (resp. $(1,2)^*$ -sint(A), $(1,2)^*$ - β int(A)) is defined to be the union of all $(1,2)^*$ - α -open (resp. $(1,2)^*$ -semi-open, $(1,2)^*$ - β -open) sets of X contained in A .

Definition 2.7

A subset A of a bitopological space (X, τ_1, τ_2) is called

(1) $(1,2)^*$ -g-closed [12, 14] if $\tau_{1,2}$ -cl(A) \subseteq U whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X . Then complement of $(1,2)^*$ -g-closed set is called $(1,2)^*$ -g-open set.

(2) $(1,2)^*$ -gsp.closed [15, 17] if $\tau_{1,2}$ - β cl(A) \subseteq U whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open. Then complement of $(1,2)^*$ -gsp-closed set is called $(1,2)^*$ -gsp-open set.

(3) $(1,2)^*$ -gs-closed [10] if $\tau_{1,2}$ -scl(A) \subseteq U whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X . Then complement of $(1,2)^*$ -gs-closed set is called $(1,2)^*$ -gs-open set.

(4) $(1,2)^*$ - \hat{g} -closed [1, 17] if $\tau_{1,2}$ -cl(A) \subseteq U whenever $A \subseteq U$ and U is $(1,2)^*$ -semi-open in X . Then complement of $(1,2)^*$ - \hat{g} -closed set is called $(1,2)^*$ - \hat{g} -open set.

(5) $(1,2)^*$ - αg -closed [2, 6] if $\tau_{1,2}$ - α cl(A) \subseteq U whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X . Then complement of $(1,2)^*$ - αg -closed set is called $(1,2)^*$ - αg -open set.

(6) $(1,2)^*$ - $\alpha \hat{g}$ -closed [2] if $\tau_{1,2}$ - α cl(A) \subseteq U whenever $A \subseteq U$ and U is $(1,2)^*$ - \hat{g} -open in X . Then complement of $(1,2)^*$ - $\alpha \hat{g}$ -closed set is called $(1,2)^*$ - $\alpha \hat{g}$ -open set.

Remark 2.8

The collection of all $(1,2)^*$ -g-closed (resp. $(1,2)^*$ -gsp.closed, $(1,2)^*$ -gs-closed, $(1,2)^*$ - \hat{g} -closed, $(1,2)^*$ - αg -closed, $(1,2)^*$ - $\alpha \hat{g}$ -closed) sets is denoted by $(1,2)^*$ -gc(X) (resp. $(1,2)^*$ -gspc(X), $(1,2)^*$ -gsc(X), $(1,2)^*$ - \hat{g} c(X), $(1,2)^*$ - αg c(X), $(1,2)^*$ - $\alpha \hat{g}$ c(X)).



Definition 2.9

A bitopological space (X, τ_1, τ_2) is called

- (1) $(1,2)^* - T_{1/2}$ -space [5, 14] if every $(1,2)^*$ -g-closed set in it is $\tau_{1,2}$ -closed.
- (2) $T_{(1,2)^*b}$ -space [2] if every $(1,2)^*$ -gs-closed set in it is $\tau_{1,2}$ -closed.
- (3) $\alpha T_{(1,2)^*b}$ -space [2] if every $(1,2)^*$ - αg -closed set in it is $\tau_{1,2}$ -closed.
- (4) $T_{(1,2)^*-\alpha\hat{g}}$ -space [2] if every $(1,2)^*$ - $\alpha\hat{g}$ -closed set in it is $(1,2)^*$ - α -closed.

Proposition 2.10 [2]

- (1) Every $\tau_{1,2}$ -closed set is $(1,2)^*$ - α -closed but not conversely.
- (2) Every $(1,2)^*$ - α -closed set is $(1,2)^*$ - $\alpha\hat{g}$ -closed but not conversely.
- (3) Every $(1,2)^*$ - $\alpha\hat{g}$ -closed set is $(1,2)^*$ - αg -closed but not conversely.
- (4) Every $(1,2)^*$ - $\alpha\hat{g}$ -closed set is $(1,2)^*$ -gs-closed but not conversely.

Remark 2.11 [11]

We have the following implication for properties of subsets

$$(1,2)^*-\alpha\text{-closed} \quad (1,2)^*\text{-semi-closed} \quad (1,2)^*\text{-}\beta\text{-closed}$$

3.BASIC PROPERTIES OF $(1,2)^*$ -(sp)*-CLOSED SETS

Definition 3.1

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(1,2)^*$ -(sp)*-closed. If $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*$ - β -open in X .

The class of all $(1,2)^*$ -(sp)*-closed subset of X is denoted by $(1,2)^*$ -(sp)* $c(X)$.

Proposition 3.2

Every $\tau_{1,2}$ -closed set is $(1,2)^*$ -(sp)*-closed.

Proof follows from the definition.



Proposition 3.3

Every $(1,2)^*$ -(sp)*-closed set is $(1,2)^*$ -gsp-closed.

Proof

Let A be a $(1,2)^*$ -(sp)*-closed. Let $A \subseteq U$ and U be $\tau_{1,2}$ -open. Then $A \subseteq U$ and U is $(1,2)^*$ - β -open and $\tau_{1,2}$ - $\text{cl}(A) \subseteq U$, since A is $(1,2)^*$ -(sp)*-closed. Then $\tau_{1,2}$ - $\beta \text{cl}(A) \subseteq \tau_{1,2}$ - $\text{cl}(A) \subseteq U$. Therefore A is $(1,2)^*$ -gsp-closed.

The converse of the above proposition is not true as seen in the following example.

Example 3.4

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Then the sets in $\{\emptyset, \{a, b\}, \{b, c\}, X\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, \{a\}, \{c\}, X\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*$ -(sp)* $\text{c}(X) = \{\emptyset, \{a\}, \{c\}, X\}$ and $(1,2)^*$ -gsp $\text{c}(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, X\}$. Clearly $\{a, c\}$ is an $(1,2)^*$ -gsp-closed but not $(1,2)^*$ -(sp)*-closed in X .

Proposition 3.5

Every $(1,2)^*$ -(sp)*-closed set is $(1,2)^*$ -g-closed.

Let A be a $(1,2)^*$ -(sp)*-closed set and U be any $\tau_{1,2}$ -open set containing A . Since A is $(1,2)^*$ -(sp)*-closed and every $\tau_{1,2}$ -open set is $(1,2)^*$ - β -open, $\tau_{1,2}$ - $\text{cl}(A) \subseteq U$. Hence A is $(1,2)^*$ -g-closed.

The converse of the above proposition is not true as seen in the following example.

Example 3.6

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Then the sets in $\{\emptyset, \{a\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, \{a\}, \{b\}, \{b, c\}, X\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*$ -(sp)* $\text{c}(X) = \{\emptyset, \{a\}, \{b\}, \{b, c\}, X\}$ and $(1,2)^*$ -gc $(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Clearly $\{a, b\}$ is an $(1,2)^*$ -g-closed but not $(1,2)^*$ -(sp)*-closed in X .

Proposition 3.7

Every $(1,2)^*$ -(sp)*-closed set is $(1,2)^*$ -gs-closed.



Proof

Let A be a $(1,2)^*$ -(sp)*-closed set and U be any $\tau_{1,2}$ -open set containing A . Since A is $(1,2)^*$ -(sp)*-closed and every $\tau_{1,2}$ -open set is $(1,2)^*$ - β -open, $\tau_{1,2}$ -scl(A) \subseteq $\tau_{1,2}$ -cl(A) $\subseteq U$. Hence A is $(1,2)^*$ -gs-closed.

The converse of the above proposition is not true in general as it can be seen from the following example.

Example 3.8

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{a, c\}, X\}$. Then the sets in $\{\phi, \{a, b\}, \{a, c\}, X\}$ are called $\tau_{1,2}$ -open and the sets in $\{\phi, \{b\}, \{c\}, X\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*$ -(sp)*c(X) = $\{\phi, \{b\}, \{c\}, X\}$ and $(1,2)^*$ -gsc(X) = $\{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Clearly $\{b, c\}$ is an $(1,2)^*$ -gs-closed but not $(1,2)^*$ -(sp)*-closed in X .

Proposition 3.9

Every $(1,2)^*$ -(sp)*-closed set is $(1,2)^*$ - \hat{g} -closed.

Proof

Let A be a $(1,2)^*$ -(sp)*-closed set and U be any $(1,2)^*$ -semi-open set containing A . Since A is $(1,2)^*$ -(sp)*-closed and every $(1,2)^*$ -semi-open set is $(1,2)^*$ - β -open, $\tau_{1,2}$ -cl(A) $\subseteq U$. Hence A is $(1,2)^*$ - \hat{g} -closed.

The converse of the above proposition is not true in general as it can be seen from the following example.

Example 3.10

Let X and τ_1, τ_2 be a defined as in example 3.6. Then $(1,2)^*$ - \hat{g} c(X) = $\{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Clearly $\{a, b\}$ is $(1,2)^*$ - \hat{g} -closed but not $(1,2)^*$ -(sp)*-closed in X .

Proposition 3.11

Every $(1,2)^*$ -(sp)*-closed set is $(1,2)^*$ - αg -closed.

Proof

Let A be a $(1,2)^*$ -(sp)*-closed set and U be any $\tau_{1,2}$ -open set containing A . Since A is $(1,2)^*$ -(sp)*-closed and every $\tau_{1,2}$ -open set is $(1,2)^*$ - β -open, $\tau_{1,2}$ - α cl(A) $\subseteq \tau_{1,2}$ -cl(A) $\subseteq U$. Hence A is a $(1,2)^*$ - αg -closed.



The following example supports that the converse of the above proposition is not true.

Example 3.12

Let X and τ_1, τ_2 be defined as in example 3.8. Then $(1,2)^*-\alpha g c(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Clearly $\{b, c\}$ is $(1,2)^*-\alpha g$ -closed but not $(1,2)^*-(sp)^*$ -closed in X .

Proposition 3.13

Every $(1,2)^*-(sp)^*$ -closed set is $(1,2)^*-\alpha \hat{g}$ -closed.

Proof

Let A be a $(1,2)^*-(sp)^*$ -closed set and U be any $(1,2)^*-\hat{g}$ -open containing A . Since A is $(1,2)^*-(sp)^*$ -closed and every $(1,2)^*-\hat{g}$ -open set is $(1,2)^*-\beta$ -open, $\tau_{1,2}-\alpha cl(A) \subseteq \tau_{1,2}-cl(A) \subseteq U$. Hence A is a $(1,2)^*-\alpha \hat{g}$ -closed.

The following example supports that the converse of the above proposition is not true.

Example 3.14

Let X and τ_1, τ_2 be defined as in example 3.6. Then $(1,2)^*-\alpha \hat{g} c(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Clearly $\{a, b\}$ is $(1,2)^*-\alpha \hat{g}$ -closed but not $(1,2)^*-(sp)^*$ -closed in X .

Proposition 3.15

Let (X, τ_1, τ_2) be a bitopological space and $A \subset X$. Then the following are true.

- (1) If A is $(1,2)^*$ -g-closed, then A is $(1,2)^*$ -gsp-closed.
- (2) If A is $(1,2)^*-\alpha g$ -closed, then A is $(1,2)^*$ -gsp-closed.
- (3) If A is $(1,2)^*$ -gs-closed, then A is $(1,2)^*$ -gsp-closed.

Proof

(1), (2), (3): Since $(1,2)^*-\beta cl(A) \subset (1,2)^*scl(A) \subset (1,2)^*-\alpha cl(A) \subset \tau_{1,2}-cl(A)$, the proof is clear.

Remark 3.16

The converse of proposition 3.15 is not true. For,

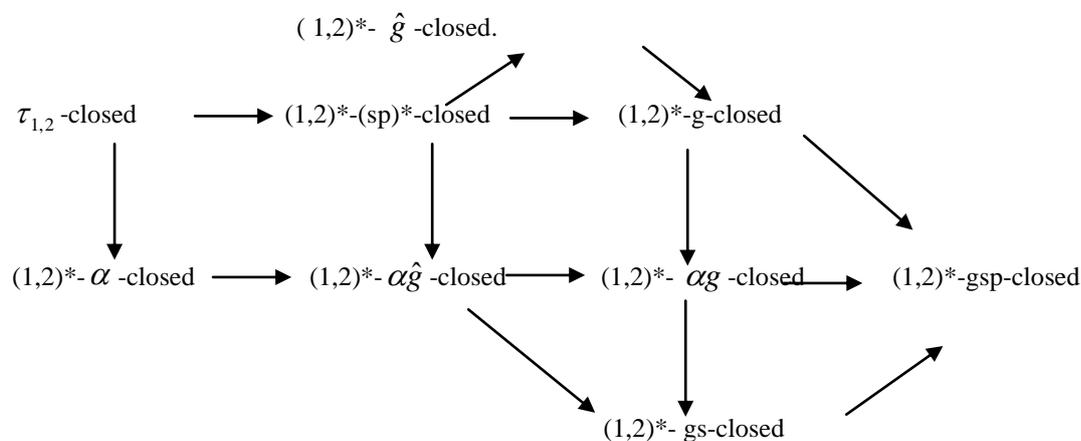


Example 3.17

Let X and τ_1, τ_2 be a defined as in example 3.4. Then $(1,2)^*\text{-gc}(X) = (1,2)^*\text{-}\alpha g c(X) = (1,2)^*\text{-gsc}(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. Clearly the set $\{b\}$ is $(1,2)^*\text{-gsp-closed}$ but it is not $(1,2)^*\text{-g-closed}$ (resp. $(1,2)^*\text{-}\alpha g$ -closed, $(1,2)^*\text{-gs-closed}$).

Remark 3.18

From the above discussions and known results in [2] we obtain the following diagram where $A \longrightarrow B$ represents A implies B , but not conversely.



None of the above implications is reversible as shown in the remaining examples and in the related paper [2].

Remark 3.19

The union of two $(1,2)^*\text{-}(sp)^*\text{-closed}$ sets need not be $(1,2)^*\text{-}(sp)^*\text{-closed}$ as shown in the following example.

Example 3.20

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$ -open and the sets in $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*\text{-}(sp)^*\text{-c}(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$. Clearly $\{a\}$ and $\{c\}$ are $(1,2)^*\text{-}(sp)^*\text{-closed}$ but $\{a, c\}$ is not $(1,2)^*\text{-}(sp)^*\text{-closed}$.

Proposition 3.21

If a set A is $(1,2)^*\text{-}(sp)^*\text{-closed}$, then $\tau_{1,2}\text{-cl}(A) - A$ contains no nonempty $\tau_{1,2}$ -closed.



Proof

Let A be is $(1,2)^*-(sp)^*$ -closed and F a $\tau_{1,2}$ -closed subset of $\tau_{1,2}-cl(A) - A$. Then $A \subset F^c$, F^c is $\tau_{1,2}$ -open and hence $(1,2)^*-\beta$ -open. Since $(1,2)^*-(sp)^*$ -closed, $\tau_{1,2}-cl(A) \subset F^c$. Consequently $F \subset \tau_{1,2}-cl(A) \cap ((\tau_{1,2}-cl(A))^c) = \phi$.

The converse of proposition 3.21 need not be true.

Example 3.22

Let X and τ_1, τ_2 be a defined as in example 3.4. Let $A = \{a, c\}$, $\tau_{1,2}-cl(A) - A$ contains no nonempty $\tau_{1,2}$ -closed set. However A is not $(1,2)^*-(sp)^*$ -closed.

Proposition 3.23

If a set A is $(1,2)^*-(sp)^*$ -closed, then $\tau_{1,2}-cl(A) - A$ contain no non empty $(1,2)^*-\beta$ -closed set.

Proof

Let A be is $(1,2)^*-(sp)^*$ -closed and S be a $(1,2)^*-\beta$ -closed subset of $\tau_{1,2}-cl(A) - A$. Then $A \subseteq S^c$ and S^c is $(1,2)^*-\beta$ -open. So $\tau_{1,2}-cl(A) \subseteq S^c$. Hence $S \subseteq (\tau_{1,2}-cl(A))^c$. Thus, $S \subseteq \tau_{1,2}-cl(A) \cap ((\tau_{1,2}-cl(A))^c) = \phi$.

Proposition 3.24

Let A be a $(1,2)^*-(sp)^*$ -closed subset of (X, τ_1, τ_2) . If $A \subseteq B \subseteq \tau_{1,2}-cl(A)$ then, B is also a $(1,2)^*-(sp)^*$ -closed subset of (X, τ_1, τ_2) .

Proof

Let U be a $(1,2)^*-\beta$ -open set of (X, τ_1, τ_2) such that $B \subseteq U$. Since $A \subseteq B$, we have $A \subseteq U$, since A is $(1,2)^*-(sp)^*$ -closed set, $\tau_{1,2}-cl(A) \subseteq U$. Also since $B \subseteq \tau_{1,2}-cl(A)$, $\tau_{1,2}-cl(B) \subseteq \tau_{1,2}-cl(\tau_{1,2}-cl(A)) = \tau_{1,2}-cl(A) \subseteq U$. Thus $\tau_{1,2}-cl(B) \subseteq U$. Hence B is also $(1,2)^*-(sp)^*$ -closed subset of (X, τ_1, τ_2) .

Proposition 3.25

If A is a $(1,2)^*-\beta$ -open and $(1,2)^*-(sp)^*$ -closed subset of (X, τ_1, τ_2) then, A is a $\tau_{1,2}$ -closed subset of (X, τ_1, τ_2) .



Proof

Since A is $(1,2)^*-\beta$ -open and $(1,2)^*-(sp)^*$ -closed, $\tau_{1,2}\text{-cl}(A) \subseteq A$. Hence A is $\tau_{1,2}$ -closed.

4. APPLICATIONS

Definition 4.1

A subset A of (X, τ_1, τ_2) is called $(1,2)^*-(sp)^*$ -open if and only if A^c is $(1,2)^*-(sp)^*$ -closed in (X, τ_1, τ_2) .

Remark 4.2

For a subset A of (X, τ_1, τ_2) , $\tau_{1,2}\text{-cl}(A^c) = [\tau_{1,2}\text{-int}(A)]^c$

Theorem 4.3

A subset A of (X, τ_1, τ_2) is $(1,2)^*-(sp)^*$ -open if and only if $F \subseteq \tau_{1,2}\text{-int}(A)$ whenever F is $(1,2)^*-\beta$ -closed and $F \subset A$.

Proof

Necessity: Let A be $(1,2)^*-(sp)^*$ -open set in (X, τ_1, τ_2) . Let F be $(1,2)^*-\beta$ -closed and $F \subset A$. Then $F^c \supseteq A^c$ and F^c is $(1,2)^*-\beta$ -open. Since A^c is $(1,2)^*-(sp)^*$ -closed, $\tau_{1,2}\text{-cl}(A^c) \subseteq F^c$. By remark 4.2 $[\tau_{1,2}\text{-int}(A)]^c \subseteq F^c$.

That is $F \subset \tau_{1,2}\text{-int}(A)$.

Sufficiency: Let $A^c \subseteq U$ where U is $(1,2)^*-\beta$ -open. Then $U^c \subset A$ where U^c is $(1,2)^*-\beta$ -closed. By the hypothesis $U^c \subseteq \tau_{1,2}\text{-int}(A)$. That is $[\tau_{1,2}\text{-int}(A)]^c \subseteq U$. By remark 4.2, $\tau_{1,2}\text{-cl}(A^c) \subseteq U$. This implies A^c is $(1,2)^*-(sp)^*$ -closed. Hence A is $(1,2)^*-(sp)^*$ -open.

Proposition 4.4

If $\tau_{1,2}\text{-int}(A) \subseteq B \subseteq A$ and A is $(1,2)^*-(sp)^*$ -open then B is $(1,2)^*-(sp)^*$ -open.

Proof

$\tau_{1,2}\text{-int}(A) \subseteq B \subseteq A$ implies $A^c \subseteq B^c \subseteq [\tau_{1,2}\text{-int}(A)]^c$. By remark 4.2, $A^c \subseteq B^c \subseteq [\tau_{1,2}\text{-cl}(A^c)]$. Also A^c is $(1,2)^*-(sp)^*$ -closed. By proposition 3.22, B^c is $(1,2)^*-(sp)^*$ -closed. Hence B is $(1,2)^*-(sp)^*$ -open.



As an application of $(1,2)^*-(sp)^*$ -closed sets we introduce the following definition.

Definition 4.5

A space (X, τ_1, τ_2) is called a $T_{(1,2)^*-(sp)^*}$ -space if every $(1,2)^*-(sp)^*$ -closed set in it is $\tau_{1,2}$ -closed.

Example 4.6

Let X and τ_1, τ_2 be defined as in example 3.6. Thus (X, τ_1, τ_2) is a $T_{(1,2)^*-(sp)^*}$ -space

Proposition 4.7

Every $(1,2)^* - T_{1/2}$ -space is $T_{(1,2)^*-(sp)^*}$ -space but not conversely.

Proof

Follow from proposition 3.5

The converse of proposition 4.7 need not be true as seen from the following example.

Example 4.8

Let X and τ_1, τ_2 be defined as in example 3.8, $(1,2)^*-\text{gc}(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Thus (X, τ_1, τ_2) is a $T_{(1,2)^*-(sp)^*}$ -space but not a $(1,2)^* - T_{1/2}$ -space.

Proposition 4.9

Every $T_{(1,2)^*b}$ -space is $T_{(1,2)^*-(sp)^*}$ -space but not conversely.

Proof

Follow from proposition 3.7

The converse of proposition 4.9 need not be true as seen from the following example.

Example 4.10

Let X and τ_1, τ_2 be defined as in example 3.6, $(1,2)^*-\text{gsc}(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Thus (X, τ_1, τ_2) is a $T_{(1,2)^*-(sp)^*}$ -space but not a $T_{(1,2)^*b}$ -space.



Proposition 4.11

Every $\alpha T_{(1,2)^*b}$ -space is $T_{(1,2)^*(sp)^*}$ -space but not conversely.

Proof

Follow from proposition 3.11

The converse of Proposition 4.11 need not be true as seen from the following example.

Example 4.12

Let X and τ_1, τ_2 be a defined as in example 3.4, $(1,2)^*-\alpha g c(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. Thus (X, τ_1, τ_2) is a $T_{(1,2)^*(sp)^*}$ -space but not a $\alpha T_{(1,2)^*b}$ -space.

Proposition 4.13

Every $T_{(1,2)^*\alpha\hat{g}}$ -space is $T_{(1,2)^*(sp)^*}$ -space but not conversely.

Proof

Let A be a $(1,2)^*(sp)^*$ -closed. Then A is $(1,2)^*-\alpha\hat{g}$ -closed. Since (X, τ_1, τ_2) is $T_{(1,2)^*\alpha\hat{g}}$ -space, A is $(1,2)^*-\alpha$ -closed. It is true that every $(1,2)^*-\alpha$ -closed is $(1,2)^*-\beta$ -closed. Therefore X is $T_{(1,2)^*(sp)^*}$ -space.

The converse of Proposition 4.13 need not be true as seen from the following example.

Example 4.14

Let X and τ_1, τ_2 be a defined as in example 3.14. Thus (X, τ_1, τ_2) is a $T_{(1,2)^*(sp)^*}$ -space but not a $T_{(1,2)^*\alpha\hat{g}}$ -space.

Theorem 4.15

For a space (X, τ_1, τ_2) the following conditions are equivalent:

- (1) (X, τ_1, τ_2) is a $T_{(1,2)^*(sp)^*}$ -space.
- (2) Every singleton subset of (X, τ_1, τ_2) is either $(1,2)^*-\beta$ -closed or $\tau_{1,2}$ -open.



Proof

(1) \rightarrow (2). Let $x \in X$. Suppose $\{x\}$ is not a $(1,2)^*$ - β -closed set of (X, τ_1, τ_2) . Then $X - \{x\}$ is not a $(1,2)^*$ - β -open set. So X is the only $(1,2)^*$ - β -open set containing $X - \{x\}$. So $X - \{x\}$ is a $(1,2)^*$ -(sp)*-closed set of (X, τ_1, τ_2) . Since (X, τ_1, τ_2) is a $T_{(1,2)^*-(sp)^*}$ -space, $X - \{x\}$ is a $\tau_{1,2}$ -closed set of (X, τ_1, τ_2) or equivalently $\{x\}$ is a $\tau_{1,2}$ -open set of (X, τ_1, τ_2) .

(2) \rightarrow (1). Let A be a $(1,2)^*$ -(sp)*-closed subset of (X, τ_1, τ_2) . Trivially $A \subset \tau_{1,2}\text{-cl}(A)$. Let $x \in \tau_{1,2}\text{-cl}(A)$. By (2) $\{x\}$ is either $(1,2)^*$ - β -closed or $\tau_{1,2}$ -open.

Case (a) Suppose that $\{x\}$ is $(1,2)^*$ -(sp)*-closed. If $x \notin A$, then $\tau_{1,2}\text{-cl}(A) - A$ contains a nonempty $(1,2)^*$ - β -closed set $\{x\}$. By proposition 3.23 we arrive at a contradiction. Thus $x \in A$.

Case (b) Suppose that $\{x\}$ is $\tau_{1,2}$ -open. Since $x \in \tau_{1,2}\text{-cl}(A)$, $\{x\} \cap A \neq \emptyset$. This implies that $x \in A$. Thus in any case, $x \in A$. So $\tau_{1,2}\text{-cl}(A) \subset A$. Therefore $\tau_{1,2}\text{-cl}(A) = A$ or equivalently A is a $\tau_{1,2}$ -closed. Hence (X, τ_1, τ_2) is a $T_{(1,2)^*-(sp)^*}$ -space.

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