A STUDY ON THE LONGITUDINAL DISPERSION WITH A REFERENCE OF SEMI INFINITE POROUS MEDIUM

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ABSTRACT

This article deals with the discussion of longitudinal dispersion in a semi-infinite porous matrix/medium. This sort of displacement has been of great concern to hydrologists who have been studying the problem of displacement of fresh water by sea water in coastal areas. The oil industry has also become involved in miscible displacement studies in connection with the possibility of flushing oil by solvents from reservoirs.

In spite of the importance of miscible displacement, its study is still in its infancy. A series of theories has been proposed for various types of flow. A great part of the discussion concerning dispersion originated in references to capillary network models for homogeneous porous media and for heterogeneous porous media.

This article, in particular, discusses the phenomenon of longitudinal dispersion process in which miscible fluids in laminar flow blend in the direction of the flow. This process is presented by regarding the cross-sectional flow velocity as time subordinate in a specific form.

The mathematical formulation of the phenomenon yields a non-linear partial differential equation. This partial differential equation is transformed into an ordinary differential equation by using one parameter amass theory of similarity analysis.

An analytical solution of the later is obtained in terms of confluent hyper geometric functions. Many researchers have discussed this phenomenon from different aspects: The solution obtained is physically consistent with the results of earlier researchers and which is also *more classical than other results obtained by various researchers.The current paper highlights the longitudinal dispersion with a reference of semi infinite porous medium.*

KEYWORDS:Longitudinal, Dispersion, Semi-Infinite, Porous

INTRODUCTION

This article manages the dialog of longitudinal scattering in a semi-unending porous matrix/medium. This kind of displacement has been of incredible worry to hydrologists who have been considering the issue of displacement of new water via ocean water in coastal areas. The oil business has additionally turned out to be associated with miscible displacement ponders regarding the likelihood of flushing oil by solvents from repositories. Regardless of the significance of miscible displacement, its investigation is still in its early stages.

A progression of speculations has been proposed for different kinds of stream. An incredible piece of the dialog concerning scattering started in references to capillary system models for homogeneous porous media and for heterogeneous porous media. This article, specifically, talks about the phenomenon of longitudinal scattering process in which miscible liquids in laminar stream mix toward the stream. This process is exhibited by with respect to the cross-sectional stream speed as time subordinate in an explicit form.

The numerical formulation of the phenomenon yields a non-direct fractional differential equation. This fractional differential equation is transformed into a common differential equation by utilizing one parameter amass theory of similitude examination. An analytical solution of the later is acquired regarding confluent hyper geometric functions. Numerous scientists have examined this phenomenon from various aspects: The solution got is physically predictable with the consequences of prior analysts and which is additionally more classical than different outcomes acquired by different specialists.

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \bar{V}) = 0 \qquad (1)$$

Where ρisthedensityfor mixture and $\bar{V}$ isthe poreseepagevelocityvector.

The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial c}{\partial t} + \nabla.\,(c\bar{V}) = \nabla.\,[\rho\,\bar{D}\nabla\,(\frac{c}{\rho})] \qquad (2)$$

Where c is the centralization of the liquid An into the other host liquid B (i.e c is the mass of A for each unit volume of the mix) and D is the tensor coefficient of scattering with nine segments. In a laminar move through a homogeneous porous medium at steady temperature ρ is consistent.

Then,

$$\nabla\,\bar{V} = 0 \qquad (3)$$

And equation (2) becomes

$$\frac{\partial c}{\partial t} + \bar{V}.\nabla\,c = \nabla.\,[\,\bar{D}\nabla c] \qquad (4.4)$$

When the seepage velocity vis along the x-axis, the non-zero components are $D_{11} = D_L$ (coefficient of longitudinal dispersion) and $D_{22} = D_T$ (coefficient of transverse dispersion) and other D_{ij} are zero. In this case the equation (2) becomes

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D_L\frac{\partial^2 c}{\partial x^2} \qquad (5)$$

Where u is the component of velocity along the x-axis which is time dependent and> 0

$$\text{Let } u(t) = u_0\, v(t) \quad (6)$$

Where is the initial velocity at a distance x.

Here, one can think about various form of v (t).We think about v (t) = exp (- mt); mt < 1 where m is the stream opposition coefficient. The scattering coefficient shifts around in direct extent to the drainage speed for different sorts of porous media Let be the underlying fixation describing the circulation of the focus at all purposes of the stream space toward the start of examination for example at t = 0. It is assumed that at first, the porous matrix isn't spotless. The time subordinate info convergence of contaminants in the porous matrix is considered at the birthplace for example x = 0 and let the focus at vast degree should be zero consistently. Subsequently, the fixation endorsed for all purposes of the boundary conditions depict the idea of communication of the stream framework with its encompassing. Subsequently, the underlying and boundary conditions can be communicated as:

$$c(x, 0) = c_i \; ; x \geq 0 \quad (7)$$

$$c(0, t) = c_0\, [1 + \exp(-qt)] \; ; t \geq 0 \quad (8)$$

$$c(x, t) = 0 \; ; t \geq 0,\, x \to \infty \quad (9)$$

Where the solute concentration and q is is the parameter like a flow resistancecoefficient.

ANALYTICALSOLUTION

The problem describing the concentration c = c(x, t) as the two miscible fluids flow through a homogeneous porous medium is given by(5). Using (6), we get $D_L = D_{O}\, v\, (t)$. Here $D_O = a\, u_o$ is the initial dispersion coefficient. Equation (5) can be written as

$$D_0 \frac{\partial^2 c}{\partial x^2} - u_0 \frac{\partial c}{\partial x} = \frac{1}{v(t)} \frac{\partial c}{\partial t}$$

The physical phenomenon of the longitudinal dispersion in a semi-infinite porous medium is mathematically represented by the following initial -boundary value problem:

$$D_0 \frac{\partial^2 c}{\partial x^2} - u_0 \frac{\partial c}{\partial x} = \frac{1}{v(t)} \frac{\partial c}{\partial t}$$

$$c(x, 0) = c_i \quad ; x \geq 0$$

$$c(0, t) = c_0 [1 + exp(-qt)] \; ; t \geq 0$$

$$c(x, t) = 0 \; ; t \geq 0, x \to \infty \tag{10}$$

We determine the analytical solution c = c(x, t) of the above mentioned initial-boundary value problem.

A new time variable is introduced by the transformation

$$\bar{t} = \int_0^t v(t)\, dt \tag{11}$$

Equation (1) becomes

$$D_0 \frac{\partial^2 c}{\partial x^2} - u_0 \frac{\partial c}{\partial x} = \frac{\partial c}{\partial t} \tag{12}$$

The initial condition (7) and boundary conditions (8) becomes and (9)

Becomes:

$$c\,(x,\,0) = c_i\ ;\ x \geq 0$$

$$c\,(0,\,\bar{t}) = c_0(\alpha - q\bar{t});\ \bar{t} \geq 0$$

$$c\,(x,\,\bar{t}\,) = 0\ ;\ \bar{t} \geq 0,\ x \to \infty \qquad (13)$$

Now, the following non-dimensional variables are introduced

$$C = \frac{c}{c_0}\ ;\ X = \frac{x\,u_0}{D_0}\ ;\ T = \frac{{u_0}^2\,\bar{t}}{D_0}\ ;\ \ Q = \frac{q D_0}{{u_0}^2} \qquad (14)$$

As a result, the above initial-boundary value problem becomes:

$$\frac{\partial^2 C}{\partial X^2} - \frac{\partial C}{\partial X} = \frac{\partial C}{\partial T} \qquad (15)$$

$$C(X,\,0) = \frac{c_i}{c_0} \qquad (16)$$

$$C(0,\,T) = c_0(\alpha - QT) \qquad (17)$$

$$C(X,\,T) = 0;\ X \to \infty \qquad (18)$$

One parameter group transformations

The procedure is initiated with the group G, a class of transformations of one- parameter ‘a’ of the form:

$$G: \begin{cases} \bar{X} = \mathbb{C}^X(a)X + K^X\ (a) \\ \bar{T} = \mathbb{C}^T(a)T + K^T\ (a) \\ \bar{C} = \mathbb{C}^C(a)C + K^C\ (a) \end{cases} \quad (19)$$

Where's and K's are real-valued differentiable functions in the real parameter 'a'.

From equations (8), we have

Now,

$$\frac{\partial \bar{C}}{\partial \bar{T}} = \frac{\partial \bar{C}}{\partial C} \cdot \frac{\partial C}{\partial \bar{T}} = \mathbb{C}^C(a) \frac{\partial C}{\partial T} \frac{\partial T}{\partial \bar{T}} = \frac{\mathbb{C}^C(a)}{\mathbb{C}^T(a)} \frac{\partial C}{\partial T}$$

$$\therefore \frac{\partial C}{\partial T} = \frac{\mathbb{C}^T(a)}{\mathbb{C}^C(a)} \frac{\partial \bar{C}}{\partial \bar{T}}$$

$$\frac{\partial \bar{C}}{\partial \bar{X}} = \frac{\partial \bar{C}}{\partial X} \cdot \frac{\partial X}{\partial \bar{X}} = \frac{\partial \bar{C}}{\partial C} \frac{\partial C}{\partial X} \frac{1}{\mathbb{C}^X\ (a)} = \frac{\mathbb{C}^C(a)}{\mathbb{C}^X(a)} \frac{\partial C}{\partial X}$$

$$\therefore \frac{\partial C}{\partial X} = \frac{\mathbb{C}^X(a)}{\mathbb{C}^C(a)} \frac{\partial \bar{C}}{\partial \bar{X}}$$

$$\frac{\partial^2 C}{\partial X^2} = \frac{\partial}{\partial X} \left(\frac{\partial \bar{C}}{\partial \bar{X}} \right) \frac{\mathbb{C}^X(a)}{\mathbb{C}^C(\ a)}$$

$$= \frac{\mathbb{C}^X(a)}{\mathbb{C}^C(a)} \frac{\partial}{\partial \bar{X}} \left(\frac{\partial \bar{C}}{\partial \bar{X}} \right) \frac{\partial \bar{X}}{\partial X}$$

$$= \frac{\mathbb{C}^X(a)}{\mathbb{C}^C(a)} \frac{\partial^2 \bar{C}}{\partial \bar{X}^2} \mathbb{C}^X(a) = \frac{[\mathbb{C}^X(a)]^2}{\mathbb{C}^C(a)} \frac{\partial^2 \bar{C}}{\partial \bar{X}^2} \qquad (20)$$

Substitution of (9) in equation (6) yields

$$\frac{[\mathbb{C}^X(a)]^2}{\mathbb{C}^C(a)} \frac{\partial^2 \bar{C}}{\partial \bar{X}^2} = \frac{\mathbb{C}^T(a)}{\mathbb{C}^C(a)} \frac{\partial \bar{C}}{\partial \bar{T}} + \frac{\mathbb{C}^X(a)}{\mathbb{C}^C(a)} \frac{\partial \bar{C}}{\partial \bar{X}} \qquad (21)$$

Equation (3.10) is said to be invariantly transformed whenever,

$$\frac{[\mathbb{C}^X(a)]^2}{\mathbb{C}^C(a)} = 1, \quad \frac{\mathbb{C}^T(a)}{\mathbb{C}^C(a)} = 1, \quad \frac{\mathbb{C}^X(a)}{\mathbb{C}^C(a)} = 1$$

$$[\mathbb{C}^X(a)]^2 = \mathbb{C}^C(a), \ \mathbb{C}^T(a) = \mathbb{C}^C(a), \ \mathbb{C}^X(a) = \mathbb{C}^C(a)$$

$$\therefore [\mathbb{C}^X(a)]^2 = \mathbb{C}^T(a) = \mathbb{C}^C(a)$$

$$\therefore \mathbb{C}^X(a) = \sqrt{\mathbb{C}^C(a)}, \ \mathbb{C}^T(a) = \mathbb{C}^C(a) = \mathbb{C}^X(a)$$

$$\therefore \mathbb{C}^C(a) = 1$$

$$\therefore \mathbb{C}^T(a) = \mathbb{C}^X(a) = 1$$

From (10), weget

$$\frac{\partial^2 \bar{C}}{\partial \bar{X}^2} = \frac{\partial \bar{C}}{\partial \bar{T}} + \frac{\partial \bar{C}}{\partial \bar{X}} \qquad \mathbf{(22)}$$

$$G: \begin{cases} \bar{X} = \sqrt{\mathbb{C}^C(a)}X + K^X\ (a) \\ \bar{T} = \mathbb{C}^C(a)T + K^T\ (a) \\ \bar{C} = \mathbb{C}^C(a)C + K^C\ (a) \end{cases}$$

The invariance of (3.11) implies (23)

2. We have C(X, 0)= $\frac{c_i}{c_0}$

$$\text{At T} = 0 \Longrightarrow \bar{T} = 0$$

$$\text{Now } \bar{C}(\bar{X},\bar{T}) = \mathbb{C}^C(a)C(X, T) + K^C(a)$$

$$\bar{C}(\bar{X},0) = \mathbb{C}^C(a)C(X, 0) + K^C(a)$$

$$= \mathbb{C}^C(a)\frac{c_i}{c_0} + K^C(a) \qquad (24)$$

We have C(0, T) = (α -QT)

$$\text{At } X = 0 \Longrightarrow \bar{X} = 0$$

$$\text{Now, } \bar{C}(\bar{X},\bar{T}) = \mathbb{C}^C(a)C(X, T) + K^C\ (a)$$

$$\bar{C}(0,\bar{T}) = \mathbb{C}^C(a)\ C\ (0, t) + K^C\ (a)$$

$$= \mathbb{C}^{C}(a)c_0(\alpha - QT) + K^{C}(a) \quad (25)$$

4. We have C(X, T) = 0; X∞

$$\text{At } X \to \infty \Longrightarrow \bar{X} \to \infty$$

Now

$$\bar{C}(\bar{X}, \bar{T}) = \mathbb{C}^{C}(a)C(X, T) + K^{C}(a)$$

$$= 0 \quad (26)$$

The initial condition (13) and boundary conditions (14) and (15) are invariantly valid if

$$K^{X}(a) = K^{C}(a) = K^{T}(a) = 0 \quad (27)$$

Thus, we have the following initial boundary valuedproblem

$$\frac{\partial^2 \bar{C}}{\partial \bar{X}^2} = \frac{\partial \bar{C}}{\partial \bar{T}} + \frac{\partial \bar{C}}{\partial \bar{X}}$$

Where $\bar{C}(\bar{X}, 0) = \mathbb{C}^{C}(a)\frac{c_i}{c_0}$

$$\bar{C}(0, \bar{T}) = \mathbb{C}^{C}(a)c_0(\alpha - QT)$$

$$\bar{C}(X, \bar{T}) = 0;\ X \to \infty \quad (28)$$

REFERENCES

1. Verma, A.P. (2010) Perturbation solution in imbibitions in a cracked porous medium of small inclination; IASH, 13, 1, 45.
2. Muskat M. (2014) Physical Principles of Oil Production; McGraw-Hill Book Co., New York, Inc, 162.
3. Gelhar, (2013) The Physics of Flow through Porous Media; University of Toronto Press, Toronto, 229.
4. Kidder, R.E. (2014) Flow of Immiscible Fluid in Porous Media: Exact Solution of a Free Boundary Value Problem, J. App. Phys., 27, 8, 867.
5. Chuoke, R.L; Van der Poel, C. (2014) The instability of slow, immiscible, viscous liquid-liquid displacement in permeable media; Pet. Trans. A.I.M.E., 216, 188.
6. Shackle, eg. (2013)Scurgerea Fluiderol Multifazice Prin Medii Porouse; Editon Acad. R.S.E. Rumania.
7. Cirpka, (2011) Calculation of linear water behavior including the effect of capillary pressure; Trans, A.I.M.E, 213.
8. McEven, C.R. (2012): A numerical solution of the linear displacement equation with capillary pressure; Trans, A.I.M.E, 216.
9. Oya and Vallochi (2014) Transient two phase capillary flow in porous media; The Phys. Fluid, 4, 5.
10. Kapoor and Gelhar, (2014) Proc. 3rd Word Petrol. Cong; The Hauge II, 544.
11. Kapoor et al., Cirpka, (2012) A theory of dispersion in a porous medium; J. Fluid Mechanics, 6, 3.
12. Kapoor and Kitanidis (2014) A unified theory for stable and unstable miscible displacement; Pet. Trans., A.I.M.E, 228, SPEJ., 205.
13. Attinger et al., Dentz et al., (2010).The stability of an interface between miscible fluids in a porous medium; ZAMP, 13, 255.