THE INTERACTION OF ENERGY WITH MATTER IN THE LIGHT OF QUANTUM MECHANICS

(STUDY ON QUANTUM MECHANICAL TREATMENT OF RADIENT ENERGY)

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ABSTRACT:

In the present work, we limit our considerations to absorption and emission phenomena where only two and three levels of the system (atoms or molecules) take part and the dispersion phenomena is dealt with at infrared frequencies. Absorption (or emission) and scattering phenomena have been investigated classically in which the individual atoms under the influence of the interacting field are treated as driven harmonic oscillators. The classical theory is adequate to describe the behavior of familiar microwave amplifiers and oscillators but shows it's inadequacy in explaining the behavior of a complex atom or a system of atoms as in maser-like devices. It is interesting to note that the T-matrices of interaction have the elements which are just the probability amplitudes and their conjugates and are recognizable as Kayley-Klein parameters which are intimately connected with spatial rotation on quantum mechanics. The T-transformation method for a single two-level particle has been extended very elegantly to treat the interaction problem for an assembly for independent or co-related two level systems. The macroscopic T-transformation operator is constructed from the product of the T- operators for different constituent atoms while the Hamiltonian operator is obtained by summing up the phase dependent Hamiltonians for the individual atoms. Other relevant macroscopic operators, namely the state selection operator, excitation and de-excitation operators, inversion operators etc. can be constructed from the individual atoms operator using the principles of quantum mechanics and statistical mechanics, while the theory for a single two-level particle conforms to the spin formulation, the theory for the assembly is seen to be based on the general angular moment formalism.

KEY WORDS: Classical Theory, Amplification process , Quantum radiation Transition probability.

INTRODUCTION:

A phenomenological treatment of the above processes as well as maser action in such systems is possible using the concepts of (I) discrete state of matter, (II) statistical distribution of the elements of matter in these states, (III) Einstein's principle of stimulated transition between these states and (IV) the relaxation mechanism in the system (namely spontaneous emission from excited states).For two - level systems, a net absorption takes place in thermal equilibrium conditions. To have a net emission, the bulk matter is to be excited such that the equilibrium population between the levels is inverted. Since two - level schemes employ the same two Eigen states which are employed in the amplification process it is clear that in maser – like devices inversion and amplification must be separated from one another in either space or time. In a time separation, the circulatory used in amplification is also employed in the inversion process but the system becomes inoperative as an amplifier, during a period at as long as the time required achieving inversion. Such systems are thus limited to pulse operation, where as in space separation, the population of an assembly is inverted in a region external to the maser circulatory proper.

These difficulties are surmounted in devices involving system with three energy levels because a steady state population inversion can be achieved between either pair of levels(namely 1-2 and 3-2) depending upon the relaxation rates, by saturating the 1-3 (pump) transitions. The semi-classical treatments, in which the material system is treated quantum mechanically and the interacting radiation as a time dependent perturbing field on the other hand, goes may steps ahead and explains many of the linear as well as non—linear response of the system qualitatively and quantitatively.

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THEORY:

The present work can define a transition rate from level 1to level 2 in the presence of a single mode field**.** It is relatively easy to make input and output calculation for a ruby laser in a hypothetical steady radiating state. Direct solutions of the differential equation are obtained from the Schrodinger equation of the problem appeared in literature and where first given by Einstein in connection with the derivation of induced transition probabilities. Literature point cut that the most widely used quantum mechanical analysis, for complete and rigorous treatment of a collection of multilevel atoms is the density matrix approach. The model may be improved proceeding along a fully quantum mechanical of the interacting systems. The interacting field should therefore be quantized and the system (atom plus radiation) be treated as a single quantum mechanical one.

ISOTROPIC RATE OF EMISSION OF POWER:

We are all familiar to the process of spontaneous emission, in which atom in an excited state E_i can emit a quantum radiation of frequency v_{ij} , there by dropping to a lower energy state E_j , according to the relation

$$E_i - E_j = h v_{ij} \quad (1)$$

where h is the Planck 's constant. These jump occur at a rate A_{ij} with a resultant spatially isotropic rate of emission of power $n_i A_{ij} h v_{ij}$, Where $n_i$is the population of atoms in the excited states. Somewhat less familiar is the concept that these same atoms can be stimulated to emit radiation v_{ij} by being bathed in radiation of the same frequency. The physical phenomenon which makes the laser possible is that of this stimulated emission of radiation. The rate of these stimulated jumps is proportional to the energy density u (v_{ij}) of the radiation and to the population difference $n_i - n_j$, between the upper and lower energy

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states. Furthermore, the stimulated radiation exhibits the same directional and polarization characteristics as that of the stimulating radiation. This is the process that gives rise to the amplification and directional properties of lasers. Einstein showed that in the steady-state, an expression of the form

$$P(\upsilon)\, \boldsymbol{B_{12}N_1} = P(\upsilon)\, \boldsymbol{B_{21}N_2} + A\boldsymbol{N_2} \qquad (2)$$

must be true to account for the transitions that take place under the influence of a broadened radiation field where P(υ) is the energy per unit volume per unit frequency interval A and B are the spontaneous and induced transition rate coefficients (or Einstein's coefficient). As a matter of fact, Einstein derived them originally, not on the basis of field quantization, but rather by the use of classical arguments and thermodynamic considerations. The expression (e.g. $\boldsymbol{B_{21}}=\boldsymbol{B_{12}}$, $\boldsymbol{A}/\boldsymbol{B_{12}}=\frac{8\pi h\upsilon^3\ \eta^3}{c^3}$) are found to be in agreement with the results obtained on the basis of field quantization.

The present work can define a transition rate from level 1to level 2 in the presence of a single mode field as,

$$\boldsymbol{\omega_{12}} = K < n > \qquad (3)$$

Whereas the transition rate from level 2 to level 1 is given by

$$\omega_{21} = K < n > + A \qquad (4)$$

As we pass from the single mode radiation case to the broad band radiation case, the equation for the population difference in terms of the transition rates still holds, but now the transition rates $\boldsymbol{\omega_{12}}$ and $\boldsymbol{\omega_{21}}$ given by (3) and (4) must be generalized by summation or integration over frequency. If the frequency distribution of the radiation field represented by < n(υ) > and the mode density P(υ) are each assumed to very slowly with respect to the line shape factor

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contained in K(υ) K = $\frac{\pi\ \omega}{\pi\ \varepsilon}$ $\frac{|\mu_{12}|^2}{3} g_L(\boldsymbol{\omega}.\ )\frac{1}{V}$, all frequency dependence other than the line shape factor may be removed from the integral. With this assumption we arrive at,

$$\boldsymbol{\omega}_{12} = \mathrm{P}\ (\boldsymbol{\upsilon}) B_{12} \quad (5)$$

$$\boldsymbol{\omega}_{21} = \mathrm{P}\ (\boldsymbol{\upsilon})\ B_{21} + \mathrm{A} \quad (6)$$

where P(υ) is the energy per unit volume per unit frequency interval defined by

$$\mathrm{P}\ (\boldsymbol{\upsilon}) = \ \mathrm{P}\ (\boldsymbol{\upsilon})\ \mathrm{h} < \mathrm{n}\ (\boldsymbol{\upsilon}) > \quad (7)$$

and $$B_{12} = B_{21} = \mathrm{B} = (\frac{1}{h\upsilon})\ \mathrm{V} \int_0^{\infty} Kd$$

with A given by $$\mathrm{A} = \frac{1}{T_{sp}} = \mathrm{V} \int_0^{\infty} K(\omega)\ P(\omega)\ dw$$

(Where $T_{sp}, K(\omega)$ and $P(\omega)\ d\boldsymbol{\omega}$ represent spontaneous emission time, single mode spontaneous emission rate and number of modes per unit volume in a frequency range $d\omega$, respectively) We see that the ratio of A to B is as follows:-

$$\frac{A}{B} = \mathrm{h}\ (\boldsymbol{\upsilon})\ \mathrm{P}(\boldsymbol{\upsilon}) = \frac{8\pi\ \mathrm{h}\ \upsilon^3\ \eta^3}{c^3} \quad (8)$$

(from the results of mode density relationships). From (7) and (8) we find

$$\frac{A}{BP} = \frac{1}{<n>} \quad (9)$$

where <n> is the expectation value of the number of photons in a single mode. From (9) we thus see that the induced transition rate B (emission or absorption) is <n> times that for spontaneous emissions.

The laser consists of vast number of atomic amplifiers placed between two partially reflecting mirrors which cause radiation to travel back and forth through the amplifying medium. The electromagnetic field, building up within the laser, may be regarded as a field in a cavity which is weakly coupled to the outside. The different types of electromagnetic oscillations of the laser regarded as an isolated cavity are the well known modes of oscillations or briefly modes. It is relatively easy to make input and output calculation for a

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ruby laser in a hypothetical steady radiating state. Such calculations are of little value, however, because of the large intensity fluctuations which seem inherent in the situation.

The power generated at frequency υ_{21}in a uniformly excited ruby laser of volume V is

$$P_0 = \omega_{21}(N_2 - N_1)\ V\ h\upsilon_{21} \quad (10)$$

Here ω_{21} is proportional to the radiation density, and $N_2 = N_1$ depends upon the radiation density as well as on the intensity of excitation. Starting with zero radiation density at frequency υ_{21} when the threshold is first reached, the radiation density start to fall again and an oscillation of intensity ensues. This oscillation is called pulsation.

1. **Transition probability from T - matrix consideration:**

For the interaction problem of radiation and two level system, it is essential to determine the complex probability amplitudes a (t) and b (t) for the two states $|a>$ and $|b>$ respectively that solve the mechanical part of the interaction problem, because, these directly (and their different real combinations r_1 *and* r_2 andr_3) give the various physical quantities of interest such as power emission from the system, polarization of the system etc. Direct solutions of the differential equation for a(t) and b(t) which are obtained from the Schrodinger equation of the problem appeared in literature and where first given by Einstein in connection with the derivation of induced transition probabilities.

It is interesting to note that the T-matrices of interaction obtained by earlier worker have the elements which are just the probability amplitudes and their conjugates and are recognizable as Kayley-Klein parameters which are intimately connected with spatial rotation on quantum mechanics.

When the system is initially in its higher state represented by the ket $|a>$, the state vector $|\psi>$, at a later time t, is essentially the transformation of $|a>$ by a matrix

$$T(t) = \begin{pmatrix} a(t) & -b^{+}(t) \\ b(t) & a^{+}(t) \end{pmatrix} \quad (11)$$

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In terms of the three well known real functions ω_1, ω_2 and ω_3 (ω'_1, ω'_2 *and* ω'_3 in the primed frame; Hamiltonians in the primed frame appear as a constant.) of Feynman equation $\frac{\overrightarrow{dr}}{dt} = (\vec{\omega} \times \vec{r})$, the interaction matrices have been expressed as for $\Delta m = \pm 1$ transitions

$$T(t) = \begin{pmatrix} \cos\frac{\Omega t}{2} - \frac{i\omega'_3}{\Omega} \sin\frac{\Omega t}{2} & \frac{-i}{\Omega}(\omega'_1 - i\omega'_2) \sin\frac{\Omega t}{2} \\ \frac{-i}{\Omega}(\omega'_1 - i\omega'_2) \sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} + \frac{i\omega'_3}{\Omega} \sin\frac{\Omega t}{2} \end{pmatrix} \quad (12)$$

with $= (\omega'^2_1 + \omega'^2_2 + \omega'^2_3)$; and for $\Delta m = 0$ transitions

$$T(t) = \begin{pmatrix} \cos\frac{\Omega t}{2} - \frac{i\omega'_2}{\Omega} \sin\frac{\Omega t}{2} & \frac{-i\omega'_1}{\Omega} \sin\frac{\Omega t}{2} \\ \frac{i\omega'_1}{\Omega} \sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} + \frac{i\omega'_3}{\Omega} \sin\frac{\Omega t}{2} \end{pmatrix} \quad (13)$$

with $\Omega = (\omega'^2_1 + \omega'^2_2)^{1/2}$

Since the above matrices refer to the rotating coordinate system (1', 2', 3') we obtain, on comparison of the elements of (11) and (13), the probability amplitudes, to be given by

$$a'(t) = \cos\frac{\Omega t}{2} - \frac{i\omega'_2}{\Omega} \sin\frac{\Omega t}{2}, \quad b'(t) = \frac{-i\omega'_1}{\Omega} \sin\frac{\Omega t}{2} \quad (14)$$

for $\Delta m = 0$ case of transition.

The transition probability $|b(t)|^2$ ($= |b'(t)|^2$) for the system (initially in the higher state) to be in the lower state after a time t due to the interaction is therefore given buy

$$|b(t)|^2 = \frac{\omega'^2}{\Omega^2} \sin^2\frac{\Omega t}{2}$$

$$= \frac{\left(\frac{\mu_{ab} E_0}{\hbar}\right)^2}{\left(\frac{\mu_{ab} E_0}{\hbar}\right)^2 + (\omega_0 - \omega)} \sin^2 \tfrac{1}{2} t \sqrt{\left(\frac{\mu_{ab} E_0}{\hbar}\right)^2 + (\omega_0 - \omega)} \quad (15)$$

(from the explicit expression for ω_1's given by Feynman et al which for a coherent applied field $E = E_0 \cos \omega t$ appear to be constants in a rotating frame obtained by rotation of the (1-2)

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plane about the 3rd axis with an angular velocity $\boldsymbol{\omega}$, $\boldsymbol{\omega}_3 = \boldsymbol{\omega}_0$ is the transition frequency of the two states). Under the resonance condition $\boldsymbol{\omega}_0 = \boldsymbol{\omega}$ the transition probability (15) changes to

$$|b(t)|^2 = \sin^2 \tfrac{1}{2} t \left(\frac{\mu_{ab} \; E_0}{\hbar}\right) \tag{16}$$

where,

$$\alpha = \tfrac{1}{2} \left(\frac{\mu_{ab} \; E_0}{\hbar}\right) \tag{17}$$

The above results are exactly identical to the results obtained for occupation probability for a two level spin system in a time harmonic field through solving Schrödinger equation.

2. Solution for the Inverted Population Difference between the Lasing Levels of a 3-levels system

Literature point cut that the most widely used quantum mechanical analysis, for complete and rigorous treatment of a collection of multilevel atoms is the density matrix approach. However, they are not very different from the approximate equations obtained with simple heuristic arguments.

The approximate approach we will develop in this section amounts in essence to treating each separate transition in a 3-level (or a multilevel) system as a separate two level transition and then adding up the various rate equation terms to find the total rate equation for each level in the system. Siegman points out that if applied signals are present at or near various transition frequencies ω_{ij} between various levels E_i and E_j, then provided that none of the applied signals is too strong, the following general principles can be applied:

(a) As far as the stimulated response on any particular transition is concerned, that transition may be treated as if it were simply an elementary two- level transition between the two energy levels E_i and E_j."

The induced response on the transition, in each case, will be proportional to the population difference ΔN on the transition and will be independent of the populations of all

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of the other energy levels, as well as independent of the presence of any allied signals on other transitions(provided they keep ΔN unchanged).

(b) "Only the populations n_i(t) of the two levels will be directly changed by the presence of an applied signal on the particular transition, and these population changes (or more precisely rates of change) can be described in the same rate equation terms as in the equivalent two- level case."

(c) A signal applied at or near a given transition frequency will excite a significant response on that transition only.

Multiple signals applied simultaneously to several different transitions in the same atomic system will not directly interact with each other."

In other words, however, leaving the indirect effects, it not the presence or absence of the other signals that counts, but simply the population difference that is present, regardless of how it is brought about.

(3) Formulation of the Problem:-

Let us suppose that, in the absence of any external radiation, the particles constituting the medium are in thermal equilibrium at temperature T. We consider that each of the N particle of the medium can exist in one of the three allowed stationary states, where Nis the number density of the particles. If n_1(t), n_2(t) and n_3(t) represent a general distribution of the particles over the states 1,2,3 with energies $E_1 < E_2 < E_3$ the rate equations can be written as

$$\frac{dn_1}{dt} = -P_{12}\, n_1 + P_{21}\, n_2 - P_{13} n_1 + P_{31}\, n_3 + \omega_{12}\,(n_2 - n_1) + \omega_{13}\,(n_3 - n_1) \quad (18a)$$

$$\frac{dn_2}{dt} = -P_{21}\, n_2 + P_{12}\, n_1 - P_{23} n_2 + P_{32}\, n_3 + \omega_{13}\,(n_1 - n_2) \quad (18b)$$

$$\frac{dn_3}{dt} = -P_{31}\, n_3 + P_{13}\, n_1 - P_{32} n_3 + P_{23}\, n_2 + \omega_{13}\,(n_1 - n_2) \quad (18\ c)$$

Where, $$n_1(t) + n_2(t) + n_3(t) = N \quad (19a)$$

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$$n_1(t = 0) = a_i \qquad (19b)$$

$P_{ij}(=\frac{1}{T_{ij}})$ and ω_{ij} are respectively the thermal and the stimulated transition probabilities per unit time between the level i and j. The relaxation processes always act to bring the energy level populations to thermal equilibrium, with a Boltzmann distribution between each pair of levels. It is reasonable to assume that in a multilevel system every atomic level population may be connected by longitudinal relaxation process of varying strength, with every other atomic energy level population, both above and below it. From physical arguments it is invariably assumed that the relaxation rate from one level $E_i$to a second level E_j will be given by a relaxation transition probability P_{ij} per atom per unit time, multiply by the number of atoms n_i available in the originating level. Of course, there will be a similar relaxation process in the reverse direction with a total rate given by$P_{ij}n_j$. At thermal equilibrium the average number of j → i thermal transitions is equal to that of i → j transitions, so that

$$\frac{P_{ij}}{P_{ji}} = \frac{n_j^e}{n_i^e} = e^{-(E_j - E_i)/kt}$$

$$= (1 - h\upsilon_{ij})/kt = \epsilon_k\text{,(say), and I,j,k=1,2,3,.....} \qquad (20)$$

The terms in the equations (18a, b, and c) are not condensed in any way, but the source of each term should be clear from the format. The relaxation terms are stated first and are followed by the stimulated terms. We have also used the fact that $\omega_{ij} = \omega_{ji}$ although this is, for course, not true for $P_{ij} and\ P_{ji}$.

Population inversion between the levels 1and 2 takes place due to the action of a pumping field at frequency υ_{13} and the relaxation processes between the levels 3and 2. The rate of growth of population inversion depends mainly upon (1) rate of pumping, (2) initial equilibrium population of the levels and (3) the rate of relaxation mechanism. As soon as the

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population of level 2 becomes greater than that of 1, downward transition takes place resulting in an emission at frequency υ_{12} and thereby increasing the energy density of the inducing field at υ_{12}.

In the present treatment we first solve for the instantaneous population inversion Δn (= $n_2 - n_1$) between the lasing levels taking ω_{12}as a function of photon density and time. As noted earlier, in connection with the analogy with a two level system with no loss, we can take $\omega_{12} \propto \sin^2 \alpha t$ (eqn. 16) where α = – ½ ($\frac{\mu_{12} \; E_0}{\hbar}$), μ_{12} being the matrix element for the component of the dipole moment along the field$\vec{E}$. In fact, n = $\sin^2 \alpha t$ is the solution of the differential equation for photon number per atom (Venkatesh and Dixit). In the later part of this chapter we solve the rate equations for the power output from the emissive system at any instant of time considering ω_{12} as time independent. The power emitted is found to vary with time which, ultimately, attains a constant value. In the microwave range the expression for power tallies with that of Bloembergen.

4 approximate solution:

By manipulating and combining the first two rate equations of (18) we have,

$$\frac{d^2 \Delta n}{dt^2} + (L_1 + 2\omega_{12})\frac{d\Delta n}{dt} + \left(M_1 + 2\frac{d\omega_{12}}{dt} + 2K\omega_{12}\right)\Delta n = \text{F} \qquad (21)$$

where Δn stands for the instantaneous difference of the populations of the lasing levels and L_1, M_1, K, F are constants involving P_{ij} and ω_{13}. The above equation (21) is of the following form is of the following form

$$\frac{d^2 y}{dx^2} + f_1(\text{c})\frac{dy}{dx} + f_2(\text{c})\text{y} = \text{C}$$

and can be solved in a suitable manner. Let us rewrite the equation as under after putting the aforesaid substitution e.g. ω_{12}= R $sin^2 \alpha t$ (R is the constant of proportionality)

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$$\frac{d^2 \Delta n}{dt^2} + (L_1 + 2\mathrm{R}\, sin^2 \alpha t\,)\frac{d\Delta n}{dt} +$$

$$(M_1 + 2RK\, sin^2 \alpha t\; + \qquad 4R\, sin\alpha t.\, cos\alpha t.\, \alpha)$$

$$\Delta n = \mathrm{F} \qquad (22)$$

On taking the trial solution of the form y=$\sum_{n=0}^{\infty}(a_0 nt)^{c+n}$ (Where $a_0 \neq 0$), we solve the above second order deferential equation (22) for the instantaneous value of the level population difference $(n_2 - n_1)$ to get

$$\mathrm{n}=a_0\left[1 - \frac{M_1}{2}t^2 - \left\{R\frac{(M_{12}E_0)^2}{6h^2} - \frac{L_1 M_1}{6}\right\}t^3 + \cdots\right]$$

$$+a_1 t\left[1 - \frac{L_1}{2}t + \left\{\frac{L_1^2}{6} - \frac{M_1}{6}\right\}t^2 + \cdots\right] + \frac{F}{2}t^2 \quad \left[1 - \frac{L_1}{3}t + \left\{\frac{L_1^2}{12} - \frac{M_1}{12}\right\}t^2 + \cdots\right] \qquad (23)$$

where,

$$M_1 = (P_{21} + P_{23} + P_{32})\,(P_{12} + P_{13} + P_{31} + 2\omega_{13}) - (P_{12} - P_{32}) \quad (P_{21} - P_{31} - \omega_{13})$$

$$L_1 = P_{21} + P_{12} + P_{31} + P_{13} + P_{23} + P_{32} + 2\omega_{13}$$

$$\mathrm{F} = \mathrm{N}[P_{12}P_{31} + P_{12}P_{32} + P_{32}P_{13} - P_{32}P_{21} - P_{31}P_{21}]$$

$$= [P_{31}P_{23} - \omega_{13}(P_{12} - P_{21} + P_{32} - P_{23})] \qquad (24)$$

For transient behavior of the population inversion we may neglect the higher powers of t and write $\Delta n = a_0 + a_1 t$. As $\omega_{12} \propto sin^2 \alpha t$ is true only for short interval of time, the linear dependence of the population inversion is valid for that interval only.

Further considering ω_{12} as time independent (steady case), we solve for the instantaneous value of the level populations from equations (18 a, b, c)

$$n_1(\mathrm{t}) = A_i + B_i \alpha t + C_i \beta t\;, \quad i = 1, 2, 3\ldots \qquad (25)$$

Where, $A_i = {}^{-\Gamma_i}/_{g}$

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$$B_i = {}^{a_i}/_2 + {}^{\Gamma_i}/_{2g} + {}^1/_D\left[-K_1 + {}^{h\Gamma_i}/_{2g} - {}^{a_i h}/_2\right]$$

$$C_i = {}^{a_i}/_2 + {}^{\Gamma_i}/_{2g} - {}^1/_D\left[-K_1 + {}^{h\Gamma_i}/_{2g} - {}^{a_i h}/_2\right]$$

$$\alpha = {}^{-\left[h-\sqrt{h^2-4g}\right]}\Big/_2$$

$$\beta = {}^{-\left[h-\sqrt{h^2-4g}\right]}\Big/_2 \qquad (26)$$

and

$$K_1 = -[(P_{21}+\omega_{21})(a_1+a_2) + (P_{31}+\omega_{13})(a_1+a_2) + a_1(P_{23}+P_{32})]$$

$$K_2 = -[(P_{12}+\omega_{12})(a_1+a_2) + 2a_2\omega_{13} + P_{32}(a_2+a_3) + a_2(P_{31}+P_{13})]$$

$$K_3 = -[(P_{13}+\omega_{13})(a_1+a_3) + P_{23}(a_2+a_3) + a_3(P_{12}+P_{21}+2\omega_{12})]$$

$$\Gamma_1 = -\mathrm{N}[(P_{21}+\omega_{21})(P_{31}+P_{32}+\omega_{13}) + (P_{31}+\omega_{13})P_{23}]$$

$$\Gamma_2 = -\mathrm{N}[(P_{12}+\omega_{12})(P_{31}+P_{32}+\omega_{13}) + (P_{13}+\omega_{13})P_{32}]$$

$$\Gamma_3 = -\mathrm{N}[(P_{13}+\omega_{13})(P_{21}+P_{23}+\omega_{12}) + (P_{12}+\omega_{12})P_{23}]$$

$$\mathrm{h} = [P_{12}+P_{21}+P_{13}+P_{31}+P_{23}+P_{32}+2\omega_{12}+2\omega_{13}]$$

$$\mathrm{g} = [(P_{21}+\omega_{21})(P_{31}+P_{13}+P_{32}+2\omega_{13}) + (P_{12}+\omega_{12})(P_{31}+P_{23}+P_{32}+2\omega_{13})$$

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$$+(P_{31}+P_{13}+2\omega_{13})\,P_{23}+P_{32}(P_{13}+\omega_{13})]$$

D = $\sqrt{h^2-4g}$ (27)

2.5 power emitted and time for maximum power output:

The power P(t), emitted from a three level system at any time t is

P(t)= $\Delta n(t)\,\omega_{12} h\upsilon_{12}$ (28)

Where $\Delta n(t)$ is the population difference at any time t between the lasing levels, and is given by

$$\Delta n(t) = n_2(\mathrm{t}) - n_2(\mathrm{t})$$

$$=(A_2-A_1)+(B_2-B_1)e^{\alpha t}+(C_2-C_1)e^{\beta t} \quad (29)$$

On substitution of the values of the constants (i.e. A, B etc.) and using the approximation that the spontaneous transition probabilities are much smaller that the induced ones, for the output power at any time t, we have,

$$\mathrm{P}(t) = \frac{N}{3}\left[P_{21}(e_3-1)-P_{22}(e_1-1)\right] h\upsilon_{12}$$

$$+\{\left[\frac{n_2^e-n_1^e}{2}\right]\cdot\left[1-\frac{\omega_{12}-\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right]$$

$$-\frac{N}{6\omega_{12}}\left[P_{21}(e_3-1)-P_{21}(e_1-1)\right]$$

$$-\left[1+\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right]\left[\omega_{12}^2+\omega_{13}^2\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}\right]\cdot t$$

$$\frac{[N-3n_2^e]}{2\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\}\quad \omega_{12}\, h\upsilon_{12}+\{\left[\frac{n_2^e-n_1^e}{2}\right]\cdot\left[1+\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right] - \frac{N}{6\omega_{12}}$$

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$$[P_{21}(e_3-1)-P_{32}(e_1-1)]\cdot\left[1-\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right]+\frac{[N-3n_2^e]}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\}$$

$$\omega_{12}\,h\upsilon_{12}-\left[\frac{\omega_{12}+\omega_{13}}{\sqrt{\omega_{12}^2+\omega_{13}^2-\omega_{12}\omega_{13}}}\right]\cdot t \qquad (30)$$

At time=0, the power output is P(t=0) = $[n_2^e-n_1^e]\,\omega_{12}h\upsilon_{12}$ and is a negative quantity, since $n_2^e < n_1^e$.

For large value of t, the power output is

$$P(t)_{large}=\frac{N}{3}[P_{21}(e_3-1)-P_{22}(e_1-1)]\,h\upsilon_{12} \qquad (31)$$

To find the time t_m, at which the output power will be maximum, we have

$$\frac{dP(t)}{dt}=[\alpha(B_2-B_1)\,e^{\alpha t_m}+\beta\,(C_2-C_1)\,e^{\beta t_m}]\,\omega_{12}\,h\upsilon_{12}\ =0\text{A}$$

or, $\quad \frac{e^{\alpha t_m}}{e^{\beta t_m}}=-\frac{(C_2-C_1)}{(B_2-B_1)}\frac{\beta}{\alpha}$

or, $\quad e^{t_m(\alpha-\beta)}=-\frac{(C_2-C_1)}{(B_2-B_1)}\frac{h+D}{h-D}$

or, $\quad t_m=\frac{1}{0.4343\,D}\log\left[\ -\frac{(C_2-C_1)}{(B_2-B_1)}\frac{h+D}{h-D}\right] \qquad (32)$

Again from the expression (31) for example,

$$P(t)_{large}=\frac{N}{3}[P_{21}(e_3-1)-P_{32}(e_1-1)]\,h\upsilon_{12}$$

we have, for microwave frequency range in steady condition,

$$\text{P (t)} = \frac{N}{3}\ [P_{21}(e^{-(E_2-E_1)/KT}-1)-P_{32}(e^{-(E_3-E_2)/KT}-1)]\,h\upsilon_{12}$$

$$=\ \frac{N}{3}\ [P_{21}\left(-\frac{h\upsilon_{12}}{KT}\right)-\ P_{32}\left(-\frac{h\upsilon_{32}}{KT}\right)]\,h\upsilon_{12}$$

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For $h\upsilon_{ij} \ll$ KT

$$= \frac{N}{3}\frac{h^2\ 12}{KT}\{P_{32}\upsilon_{32} - P_{21}\upsilon_{21}\} \quad (33)$$

This is exactly the same result as was obtained by Bloembergen.

DISCUSSION:

The semi-classical model explains many laser phenomena, both linear and nonlinear, qualitatively and in several respects. The inadequacy result mainly due to the model itself which embodies the material system and the interacting field as two separate systems and as such cannot give appropriate description of the collective radiation phenomena discussed by Dicke and Senitzky. Besides, it is not clear from the usual S.C.F.A (self consistent field approximation) derivations just how the effects of spontaneous emission should be taken into account. These effects have important bearing on coherence properties of laser such as line width, amplitude and intensity fluctuation. Further the model sheds almost no light on the important question of statistical nature of laser radiation. The traditional approach to this problem has been the phenomenological one of adding suitable source terms based on equilibrium or other considerations to the semi-classical equations of motion. Lamb, Bloembergen and several other workers have work along these lines.

The model may be improved proceeding along a fully quantum mechanical of the interacting systems. The interacting field should therefore be quantized and the system (atom plus radiation) be treated as a single quantum mechanical one.

Distinctive approaches for the problem are in use under the semi-classical approximation among which density matrix method of Lamb, Bloembergen and others; Gyroscopic method of Feynman et al, intuitive approach of Dicken and T-matrix method of Venkatesh et al., are important because of their wide range of applicability's on one hand and for providing physical picturization of the process on the other. Senitzky's treatment of loss mechanism in such systems bears much importance in the discussion of noise sources of laser radiation,

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line broadening etc. Feynman on the other hand transformed the amplitude equations for two – levels system to give a geometrical picture of the interaction process in an abstract space and at the same time a solution for the physical quantities of interest directly from the geometrical picture itself. However for a systematic study of the problem, an analytical solution without any reference to a geometrical representation of the process is always desirable.

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