ANALYSIS OF THERMAL STRESSESAND MECH. LOADING ON FGM COMPOSITE BEAM BY MATLAB TOOL

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Abstract

Functionally graded materials (FGM) are most commonly used for barrier coating against large thermal gradient & simplification reduces modeling complexity and computation requirements but sacrifices the accuracy of through the thickness information. Now a day's FG materials are replacing the composite materials because in high temperature environment various discontinuities like cracks, debonding, delamination etc. are accounted in composite material. In FGM, variation of material properties are continues across the thickness.Then micromechanical modeling of functionally graded thermal barrier coating is considered to predict stresses under thermal and mechanical loading. In mechanical loading uniformly distributed load is subjected to FG cantilever beam and in thermal loading temperature difference is used for obtaining the axial stress results and then compared with the previous research work. Thermo-mechanical stress distribution for a three layered FGM composite beam having a middle layer of FGM is obtained by analytical method. The performance is evaluated by taking young's modulus as per power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM) across the thickness. The mathematical tool MATLAB is employed for generating the code.

Keywords: FGM Residual stresses ,mechanical loading & Thermal stress.

THEORY

1.1 Motivation

In recent time composite material are changed in functionally graded materials (FGMs). Which are advanced multiphase composites and have a smooth spatial variation in material. Functionally graded materials (FGMs) are made from a chemical-alloy mixture of metals and ceramics. FGMs are useful for many engineering sectors such as the aerospace, aircraft, automobile, and defense industries, spring and most recently the electronics and

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biomedical sectors [1]. A functionally graded material (FGM) is made from metal & ceramic. Ceramic have mechanically brittle and good high-temperature behavior.

1.2DrawbacksofLaminatedComposites

The laminated composite materials provide the design flexibility, stiffness and strength. The anisotropic constitution of laminated composite structures often result in stress concentrations near material and geometric discontinuities that can damage in the form of matrix cracking and adhesive bond separation. FGMs alleviate these problems because of a continuous variation of material properties from one surface to other.

1.3 FGMs Applications

AwidevarietyofapplicationsexistforsmartFGMstructures.Aerospace,Engineering,Nuclearenergy,Optics,Electronics, Bone, Biomaterials.

1.4 Research Goal

The material (FGM) properties are usually continuous variation in one direction. So the temperature distribution used in several applications such as nuclear reactors, ovens, space shuttles, aircrafts and combustion chambers.. The aim of this research is to determine the thermal and normal stresses generation and deflection in neutral axis of FGM materials which is substitute of traditional materials. The study will focus on the modeling and imitation of:

1. Functionally graded beam structures with material properties varying throughout the thickness of the beam.
2. Relationship & graph generation between according to variation of thickness with different property of the material. Example: Residual Stresses, thermal expansion, modulus of elasticity, modulus of rigidity, thermal conductivity, poison ratio etc.
3. Thermal gradient due to one-dimensional through-thickness steady heat conduction is considered.
4. The material properties are taken from literature which having a smooth temperature variation usually in one direction heat flow in FGM for Different material. Examples SIC- C, Al_2O_3 – Steel or Al_2O_3 – (W, Ti)C.
5. Analysis on Elastic thermo-mechanical stresses in FGM structures and thermal modeling with different temperature.

Studies are doing on the static and dynamic thermo-elastic behavior of FGM beams, cantilever or beam-like structures and Mathematical Analysis or Mat-Lab Formulation on thermal stress , thermo-mechanical loads behavior on different martial

2. CALCULATION :-

2.1 FGM Material Structure Composition

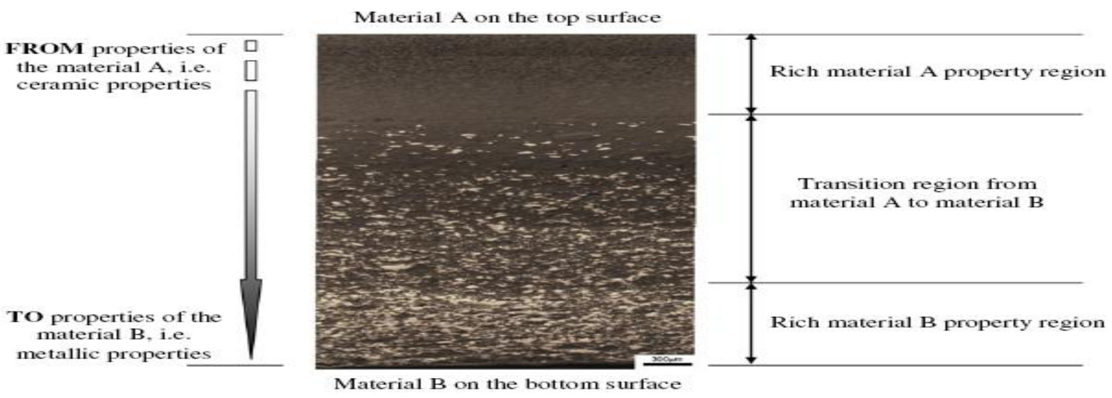

Figure. 2.2a Illustration of the FGM concept by means of microphotography for FGM [1].

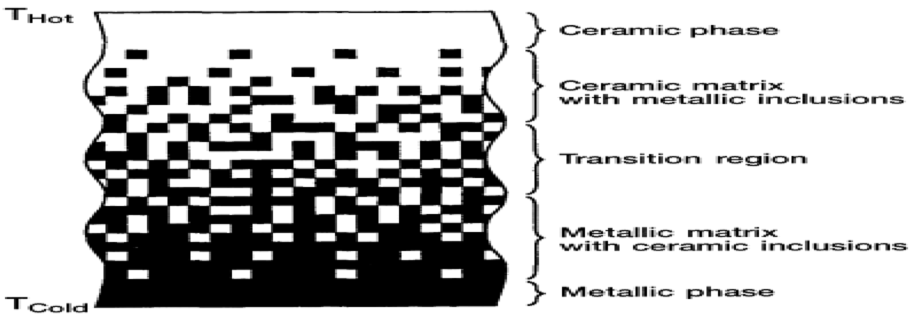

Figure 2.2b Graphical FGM Representation of Gradual Transition in the Direction of the Temperature Gradient

3.Calculation :-

3.1 Volume fraction distribution law's of FGMs

In Power Law (P-FGM), a model is created that describes the function of composition throughout the material. In Figure 3.3b, the volume fraction V_c, describes the volume of ceramic at any point z across, the thickness h according to a parameter n which controls the shape of the function [2].

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{3.1}$$

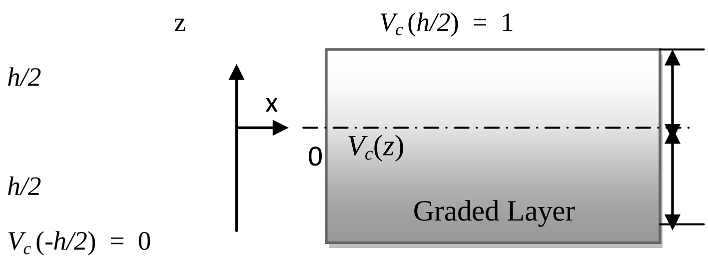

Figure 3.3b Ceramic Volume Fractions Across the FGM Layer

In the law of FGM denoted the volume fraction of metal, $V_m(z)$, in the FGM is $1\text{-}V_c(z)$. A graphical representation of volume fraction of ceramic for various values of the parameter n can be seen in Figure 3.4.

The area to the right of each line represents the amount of metal, and the area to the left represents the amount of ceramic in the material. It should be noted that $n \rightarrow 0$, the material approaches to a homogeneous ceramic, while as $n \rightarrow \infty$, the material becomes entirely metal. For $0 < n < \infty$, the metal will contain both metal and ceramic. When $n = 1$, the distribution of ceramic and metal is in equal portion. According to Nakamura and Sampath [3], the values of n should be taken in the range of **(1/3, 3),** as values outside this range will produce an FGM having too much of one phase.

3.2 Effective Properties of FGM

Effective properties of FGM are obtained by basic three laws i.e. Power Law (P-FGM), Exponential Law (E-FGM) and Sigmoid Law (S-FGM).

Table 3.1 Effective property formulas of FGMs [57]

Material property	Property related formula
Thermal conductivity (k)	$k(z) = k_t\left(1 + \frac{3(k_b - k_t)V_m(z)}{3k_t V_m(z) + (k_b + 2k_t)V_c(z)}\right)$
Modulus of elasticity(E)	$E(z) = E_t\left(\frac{E_t + (E_b - E_t)(V_c(z))^{2/3}}{E_t + (E_b - E_t)[(V_c(z))^{2/3} - V_c(z)]}\right)$
Poission's ratio (v)	$v(z) = (v_t - v_b)V_c(z) + v_b$
Coefficient of thermal expansion (α)	$\alpha(z) = (\alpha_t - \alpha_b)V_c(z) + \alpha_b + \left(\frac{V_m(z)V_c(z)(\alpha_t - \alpha_b)(k_b - k_t)}{(k_b - k_t)V_c(z) + k_b + (3k_b k_t/4G_m)}\right)$
Density (ρ)	$\rho(z) = (\rho_t - \rho_b)V_c(z) + \rho_b$
Yield strength (σ_y)	$\sigma_y(z) = (\sigma_{yt} - \sigma_{yb})V_c(z) + \sigma_{yb}$

In Table 3.1, K and G are the bulks modulus and modulus of rigidity, respectively. Also, the undefined parameters are given by

$K_t = \frac{E_t}{3(1-2V_t)}$; $G_t = \frac{E_t}{2(1+V_t)}$ $G_b = \frac{E_b}{2(1+V_b)}$; $K_b = \frac{E_b}{3(1-2V_b)}$

The subscripts t and b stand for the material property at the top and bottom, respectively for the corresponding property. t corresponds to the material property of the pure ceramic, and b corresponds to the material property of the pure metal

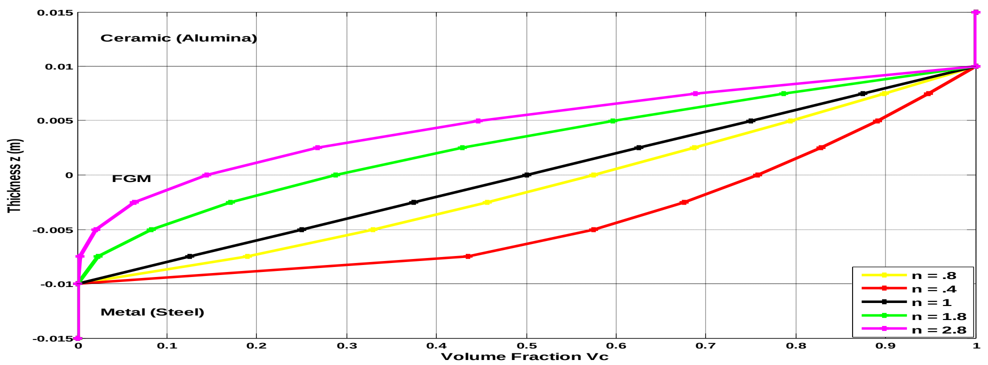

Figure 3.4 (a) Effect of Power Law Index (n) on the Volume Fraction

One of most common methods to determine the effective properties of FGM is the rule of mixtures and is given by

$$P(z) = (P_t - P_b)V_c(z) + P_b \qquad (3.2)$$

$$E(z) = (E_t - E_b)V_c(z) + E_b \quad (3.3)$$

4. FORMULATION OF GOVERNING EQUATIONS

4.1.1 One-dimensional Heat Conduction Steady-State Exact Solution for 3-Layer FGM beam This part considers the solution of the conduction steady-state problem in a composite beam consisting of 3 layers, which are assumed to be in prefect thermal contact. Figure 4.1 show the geometry coordinates and boundary condition for this problem. This section is a formulation to find the one dimensional temperature distribution for a 3-layer beam with a middle FGM layer.

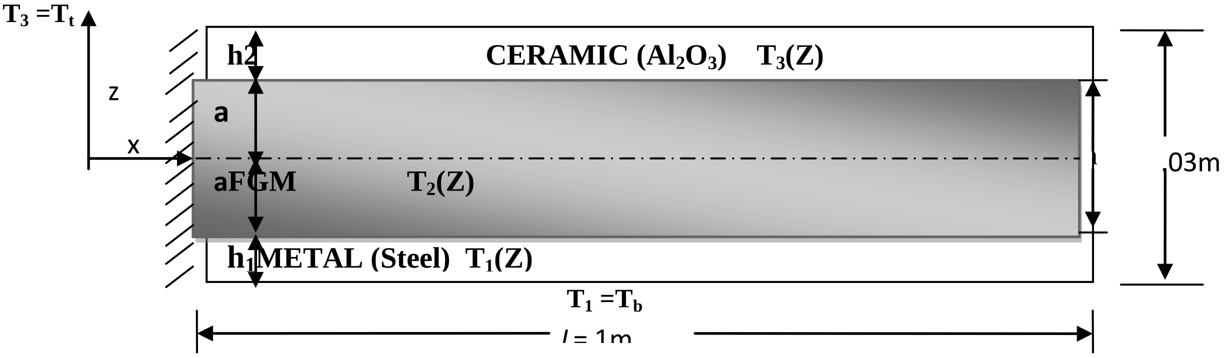

Figure 4.1 Three layer beam with perfect thermal contact at the interface surface

The mathematical formulation of this problem is given with boundary condition as

$$\frac{d}{dz}\left[\frac{k_1 dT_1(z)}{dz}\right] = 0, \qquad -(h_1 + a) < z < -a \qquad 4.1$$

$$\frac{d}{dz}\left[\frac{k_2 dT_2(z)}{dz}\right] = 0, \qquad -a < z < a \qquad 4.2$$

$$\frac{d}{dz}\left[\frac{k_3 dT_3(z)}{dz}\right] = 0, \qquad a < z < (a + h_2) \qquad 4.3$$

Subject to boundary and interface condition

$$T_1 = T_b \quad \text{at} \quad z = -(h_1 + a) \qquad 4.4$$

$$\left.\begin{aligned} \frac{k_1 dT_1(z)}{dz} &= \frac{k_2 dT_2(z)}{dz} & 4.5 \\ T_1 &= T_2 & 4.6 \\ \frac{k_2 dT_2(z)}{dz} &= \frac{k_3 dT_3(z)}{dz} & 4.7 \end{aligned}\right\} \text{at} \quad z = -a$$

$$T_2 = T_3 \quad \text{at} \quad z = a \qquad 4.8$$

Where k_1, k_2 and k_3 are the thermal conductivity coefficient for metal (steel), graded layer, and ceramic (alumina).the solution to the equation (4.1-4.3) subjected to the boundary and interface condition given by Eqs.(4.4-4.8) can be found the numerically. In Special cases can results in exact solution such as when $k_1 = k_b$ and $k_3 = k_t$ are constant throughout layers 1 and 3, while $k_2(z)$ is assumed to vary only in direction of the beam thickness

$$k_2(z) = k_t e^{-.5\ln\left(\frac{k_t}{k_b}\right)\left(1-\frac{z}{a}\right)}$$

4.9

The solution of the ordinary differential equation (4.1-4.3) for each layer is given in form

$T_1(z) = C_1 z + C_2$

4.10

$T_2(z) = C_3 \left(\frac{k_t}{k_b}\right)^{-.5z/a} + C_4$

4.11

$T_3(z) = C_5 z + C_6$

4.12

4.2 Beam Theory for Stress Calculations

. Beam is subjected to uniformly distributed transverse loading with continuous and smooth grading of metal and ceramics based on P-FGM Law, E-FGM Law and S-FGM Law are considered for study and Poisson ratio is to be held constant across FGM layer..The dimension of FGM beam of width unity and thickness *h* are considered, where the material property varies continuously in thickness direction (z). FGM beams have their volume fraction of ceramics V_c defined according to the power law function, sigmoid law function and exponential law function and the volume fraction of metal V_m is obtained as

$$V_m(z) = 1 - V_c(z) \quad (4.13)$$

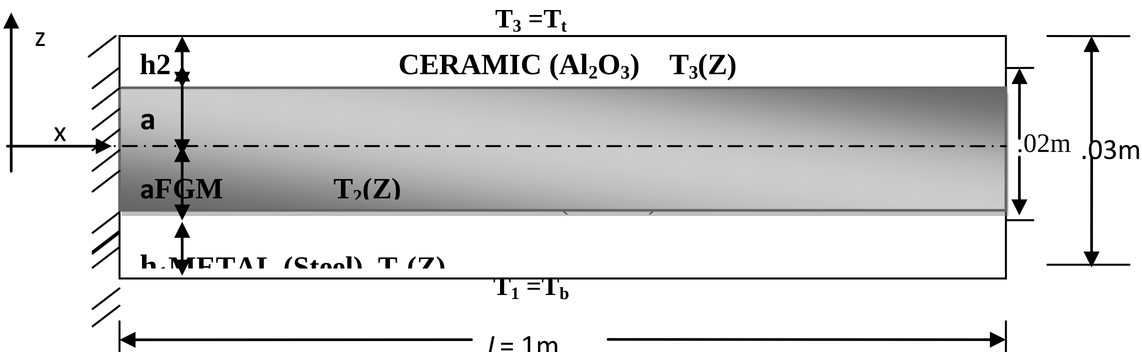

Figure 4.2 Three Layer Composite FGM Beam under Distributed Load

The mathematical modeling for evaluating the properties of functionally graded materials (P(z)) or P_b is the bottom layer property and P_t is the top layer property which are chosen from any of the three laws expresses as per the Equations 3.2, 3.7, 3.9-3.11.

$$P(z) = P_b + (P_t - P_b)V_c(z) \quad (4.14)$$

The basic assumptions are derived by that laws which is:

1. The beam is assumed to be in a state of plane strain, it is normal to the xz plane.
2. Euler-Bernoulli type beam theory is applied.
3. There is no variation in thickness along the length of beam.
4. Poisson's ratio is to be held constant along FG layer.
5. Material properties are independent of temperature gradient.

For a cantilever beam, the displacement field can be written as [51]:

$$w(x, z) = w(x)$$
$$u(x, z) = u_0(x) - z\frac{dW(x)}{dx}$$

In above equations, u and w are denoted as horizontal and vertical displacement of beam across the thickness. It may be noted that u_0 denotes displacement of points on the middle surface of the beam along the x direction. It is assumed that σ_{zz} is negligible. Then the stress-strain relations take the form:

$$\sigma_x(z) = \breve{E}(z)\varepsilon_x, \ \tau_{xz}(z) = \breve{G}(z)\gamma_{xz} \quad (4.15)$$

Where the plane strain Young modulus is given by:

$$\breve{E} = \frac{E}{1-\upsilon^2}$$

The expressions for axial strain and stress can be derived as:

$$\varepsilon_x = \frac{du(x,z)}{dx} = \frac{d}{dx}\left(u_0(x) - z\frac{dW(x)}{dx}\right) = \varepsilon_{x0} + zk_x$$

$$\varepsilon_{x0} = \frac{du_0}{dx} \quad , \quad k_x = -\frac{d^2w(x)}{dx^2}$$

$$\sigma_x(z) = \breve{E}(z).\varepsilon_{x0} + z.\breve{E}(z).k_x \quad (4.16)$$

$$\begin{bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{13} & 0\\ Q_{13} & Q_{33} & 0\\ 0 & 0 & Q_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{bmatrix} \quad (4.17)$$

$$[\bar{Q}_{ij}] = [Q_{ij}]$$

$$Q_{11} = \frac{E}{1-\upsilon^2} = Q_{33}, \ Q_{13} = \frac{\upsilon E}{1-\upsilon^2} \quad , \quad Q_{55} = \frac{E}{2(1+\upsilon)}$$

Here, $[\bar{Q}_{ij}]$ and $[Q_{ij}]$ both are stiffness matrices and ε_{x0}, k are axial strain in the middle surface and the beam curvature. According to Euler-Bernoulli beam theory, the axial force and bending moment, N and M, are defined

$$(N, M) = \int_{-h_1}^{h_1}[\breve{E}(z).\varepsilon_{x0} + z.\breve{E}(z).k](1, z)dz \quad (4.18)$$

$$C_0\varepsilon_{x0} + C_1k = 0$$

$$C_1\varepsilon_{x0} + C_2k = M_{maximum}$$

C_0, C_1, and C_2 are the coefficients of mid-plane strain and curvature. Using Equation 4.16, the axial stresses in ceramic, metal and FGM section across the thickness of proposed model are obtained.

4.3Temperature Profile modeling for thermal stress formulation

When proposed FGM beam model is subjected to uniform temperature change (ΔT), the total strain under a small strain assumption, can be taken as made up of elastic and thermal part. For a beam under plane strain condition, the only non-zero stress component is σ_x [4]:

$$\sigma_x = E(z)[\varepsilon_{x0}^T + z.k^T - \alpha(z)\Delta T] \quad (4.19)$$

Where ε_{x0}^T is the strain at the mid-plane (z = 0) of the FGM layer and k^T is the laminate curvature due to temperature gradient. Since only thermal loading is considered here:

$$\sum F_x = 0 \quad , \quad \sum M_x = 0$$

On the other hand $(N, M) = \int_{-h_1}^{h_1} \sigma_{xx}(1, z)\,dz = 0$

The axial force and bending moment in thermal gradient can be obtained as given below:

$$N^T = (\Delta T)\sum_{k=1}^{m}[\bar{Q}]_k[\alpha]_k(h_k - h_{k-1})$$

(4.20)

$$M^T = \frac{1}{2}(\Delta T)\sum_{k=1}^{m}[\bar{Q}]_k[\alpha]_k(h_k^2 - h_{k-1}^2) \quad (4.21)$$

Here, m is the number of lamina and in proposed model three laminas is considered. Further thermal strain, mid-plane strain and curvature, mechanical strain and thermal stresses are calculated by below formulas:

$$\{\varepsilon^T\} = (\Delta T)\{\alpha\}$$

$$\begin{bmatrix}\varepsilon_{x0}^T \\ k^T\end{bmatrix} = \begin{bmatrix}A & B \\ B & D\end{bmatrix}^{-1}\begin{bmatrix}N^T \\ M^T\end{bmatrix}$$

$$\{\varepsilon\} = \{\varepsilon_{x0}^T\} + z\{k^T\}$$

$$\{\varepsilon^M\} = \{\varepsilon\} - \{\varepsilon^T\}$$

$$\{\sigma^T\} = [\bar{Q}]\{\varepsilon^M\}$$

(4.22)

The coefficient of thermal expansion for FGM is obtained by rule of mixture

$$\alpha(z) = (\alpha_c - \alpha_m)(V_c) + \alpha_m \quad (4.23)$$

5. PERFORMANCE EVALUATION

5.1.1 Axial stresses

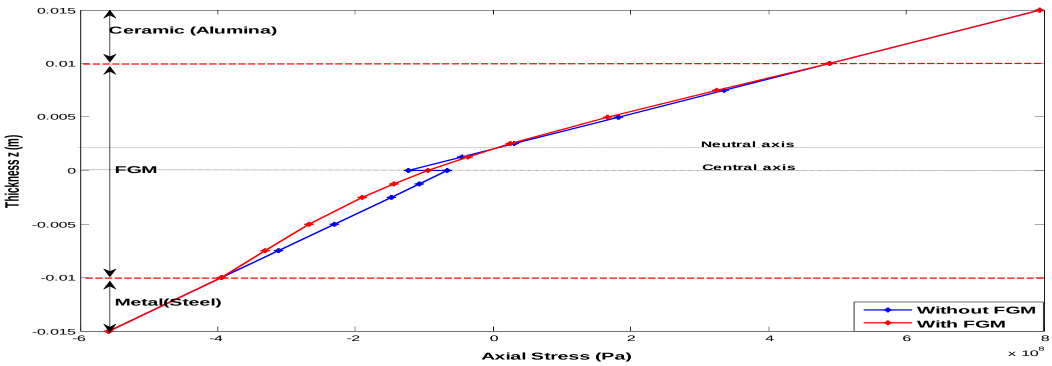

Figure 5.1(a) Axial stress with FGM & without FGM (present Work)

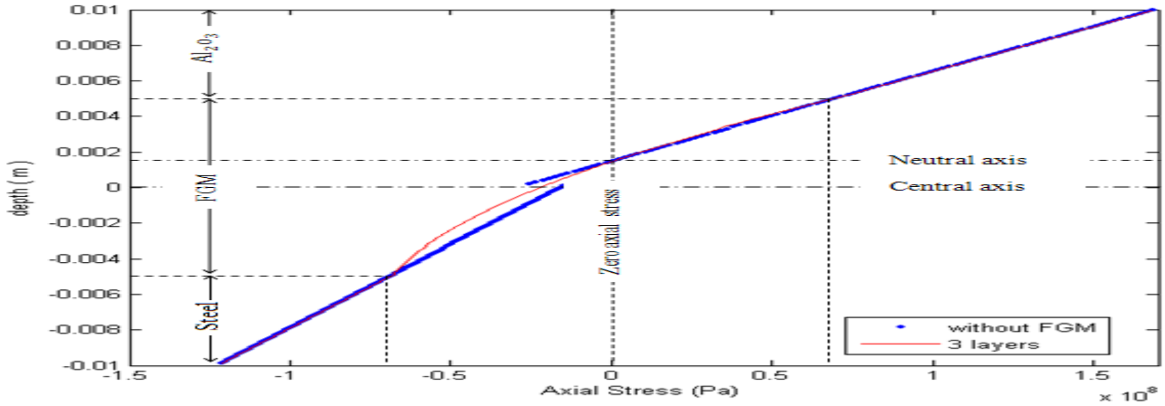

Figure 5.2(a) Axial Stress Distribution with FGM Beam, (b) Reprinted From ref. [61]

5.3 Comparison with Simon Caraballo Model (*Thermal Loading*)

In second group of comparison three-layered composite system of Al_2O_3-FGM-Steel as shown in Figure 5.3 is subjected to thermal loading only ($\Delta T=200^oC$). Thermal stresses are obtained across the thickness and are compared with the Simon Caraballo Model. Variation of thermal expansion coefficient across the thickness obtained by rule of mixture.

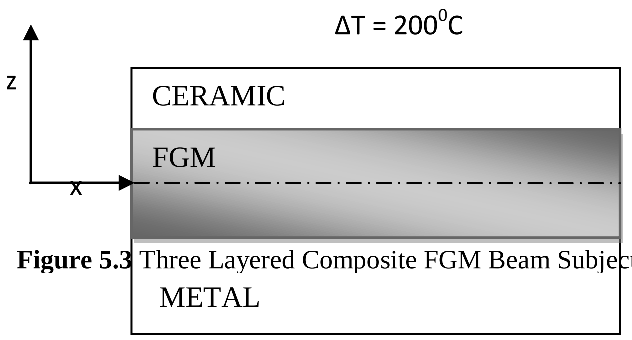

Figure 5.3 Three Layered Composite FGM Beam Subjected to Thermal Loading

The axial thermal stresses are calculated using Equation 4.22 across the thickness and are given . The axial thermal stress distribution for two-layered and three layered composite beam is presented in Figure 5.4 and the observations can be summarized as

1. When there is no graded layer between the ceramic and metal layer, large values of stresses are developed at the interface.
2. The near-interface region of the metallic layer is in tension, while the corresponding region for the ceramic layer is in compression (there is considerable abrupt change in magnitude and sign of the stress at the interface).
3. When a graded interlayer is introduced, the magnitude of the stress at the interface can be significantly reduced and the abrupt change in the stress sign is eliminated.

4. The stresses vary linearly with z within the metallic and ceramic layer, and approximately parabolically within the functionally graded layer

From these trends, it can be concluded that the results obtained in this work reasonably matches with that of Simon Caraballo model [62]. It can also be seen from above studies, that FGM layer plays more important under thermal loading.

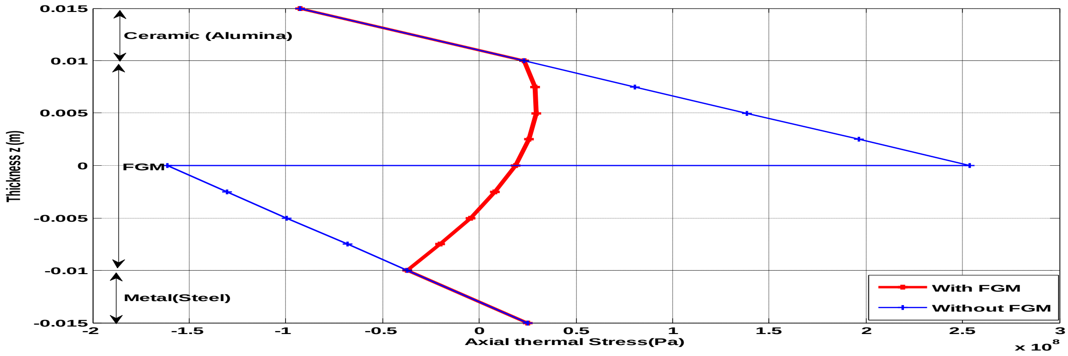

Figure 5.3(a) Axial Thermalstress with FGM & without FGM (present Work)

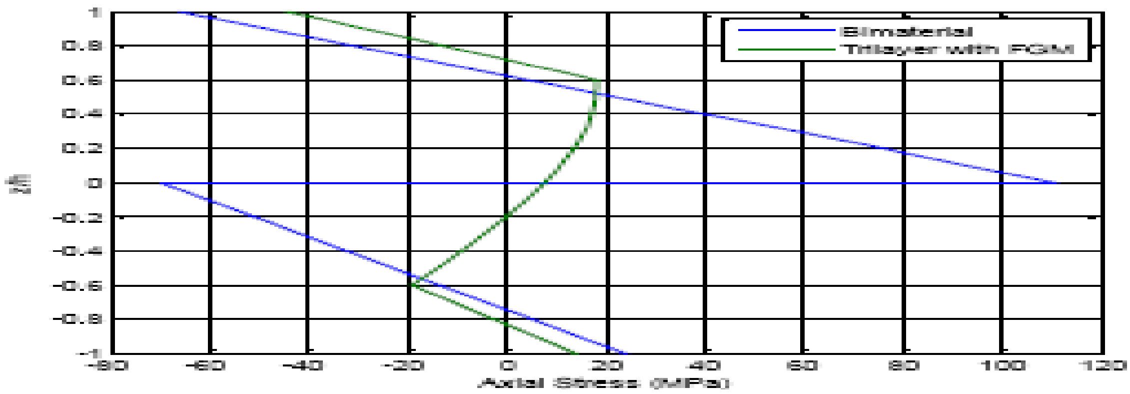

Figure 5.4(a) Axial Thermal Stress Distribution in FGM Beam, (b) Reprinted From ref [5].

Conclusions

Functionally graded materials are good replacement of composite materials because they overcome the debonding type problems. These materials are commonly used in aerospace industries where the harsh temperature is major issue. The basic properties of FGM can be obtained by any of the three function laws, power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM).

In the present work the beam model simulation with temperature distribution is taken, distribution of temperature profile in one dimensional analysis in different FGM composite material for high heat insulation property & explore the effects of spatial temperature variation in the axial and through the thickness and compare three FGM composite and find out alumina and carbon (W,Ti) has low residual stresses than the other two. So for designing purpose this FGM composite is better than the other two.The axial stresses in FGM beam under uniformly distributed load and the residual stresses due to temperature drop during curing of the FGM composite or temperature rise in operation are calculated analytically by thermal stress due to thermal loading. These results are found to be compared with the different function law (P-FGM, S-FGM & E-FGM) & previous work.

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