



VIBRATIONAL ANALYSIS OF FGM CONICAL SHELL

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ABSTRACT

The main objective of this study is to allow a thorough review of the literature on the vibrational analysis of functionally graded conical shells and functionally graded truncated conical shells. The FGM conical shell has wide area of application in various engineering fields such as civil, mechanical, petroleum, marine, military etc. Different types of theories and methods are being used in the design of FGM conical shells are investigated in this paper.

Keywords: Functionally graded conical shell, Vibrational Analysis.

I. INTRODUCTION

The composite materials were as traditional material in many engineering fields, but it was not stable at high temperature conditions. Later a ceramic material was used in place of composite material. It was more stable at high temperature conditions than composite materials but it mechanically unstable due to low toughness. To get rid of this instability, the new material has been invented in 1984 in Japan called as Functionally Graded Material(FGM). It can withstand at high temperature conditions upto 2000^oc and specially designed for aerospace industries.

The FGM are referred as a material with changing porosity, microstructure, and composition through the volume of material(1). To perform a set of various functions, the FGM were designed with changing properties over the volume of bulk material on the basis of their area of application in which they can used(2-4). There are different types of FGM available include:

- Chemical composition gradient FGMs
- Porosity and pore size gradient-structured FGMs
- Microstructural gradient-structured FGMs



In chemical composition gradient FGMs, the chemical position is varied gradually in accordance with spatial position in the material. This is available in two forms namely: single phase, or in multiphase materials(5)

The porosity in the material change with the position in the whole material in porosity gradient-structured FGM(6-7). This type of FGMs are widely used in biomedical applications because it helps in the integration of implants and tissues, and also improves the blood circulation to integrated tissues(8-9).

The microstructural gradation can be attained in solidification process during heat treatment. The microstructural gradient FGMs use in those components having hard surface to avoid wear and high toughness to resist impact that occurs during operation(10-11).

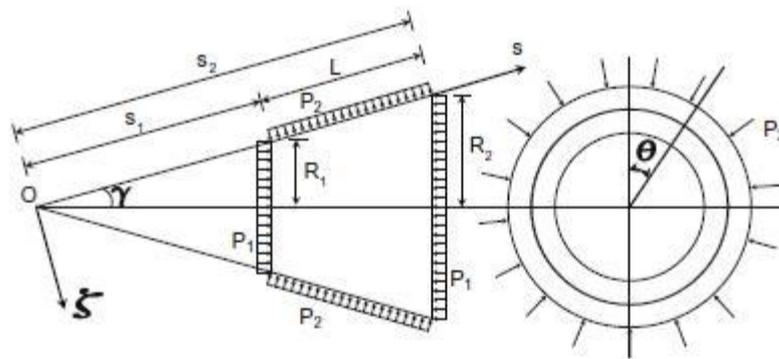


Fig. 1. Geometry of the conical shell.

II. LITERATURE REVIEW

A.H. Sofiyev[12] developed a vibration and stability behaviour of freely supported FGM conical shells subjected to external pressure by using Galerkin method. The properties of functionally graded material were assumed to vary continuously through the thickness in accordance with Donnell's shell theory.

First order shear deformation theory [FSDT] was adopted by X. Zhao and K.M. Liew[13] to analyse the free vibration analysis of functionally graded conical shell panels using a meshless method. The material properties of the conical shell panels were assumed to vary in accordance with a power-law distribution through their thickness. The first-order shear deformation shell theory was used to account for the transverse shear strains and rotary inertia. The results are obtained by using Ritz method and the equations were solved by general differential quadrature method [GDQM].

A.R. Setoodeh et al[14] developed a transient dynamic and free vibration analysis of functionally graded truncated conical shells with non-uniform thickness subjected to mechanical shock loading by using Hamilton's principle. The differential quadrature method was used to discretize the resulting equations in the axial direction. DQM or Newmark's time integration scheme (named LWDQN) was employed to solve the developed time-dependent equations. The material properties are graded continuously in the thickness direction according to a volume fraction power-law distribution.

Three-dimensional free vibration analysis of functionally graded truncated conical shells subjected to thermal environment was investigated by A.R. Fiouz and M. Subhrouyan[15] by using first order shear



deformation theory and Donells shell theory. The material properties were assumed to be temperature dependent in the radius direction, and were varied according to a simple power law distribution

Galerkin method and Runge-Kutta methods were adopted by M.R. Eslami et al[16] for the linear thermal buckling analysis of truncated hybrid FGM conical shells. The material

properties of functionally graded conical shells were assumed to vary continuously through the thickness direction based on a power law form . FSDT, Donells shell theory , classical shell theory and the Sanders nonlinear kinematics relations were used.

Free vibration analysis of rotating functionally graded truncated conical shells was investigated by P. Malekzadeh and Y. Heydarpour[17]. The initial dynamic equilibrium equations of motion and the related boundary conditions were derived based on the first-order shear deformation theory (FSDT) of conical shells. The material properties were assumed to be graded in the thickness direction. The differential quadrature method was used to obtain the natural frequencies.

Using three-dimensional (3-D) elasticity theory, A.R. Setoodeh et al[18] presented three-dimensional transient analysis of functionally graded truncated conical shells with variable thickness subjected to an asymmetric dynamic pressure. The FG conical shell is graded in the radial direction. A hybrid method composed of the layerwise theory, differential quadrature method (DQM), and Fourier series expansion was employed for the analysis

Nguyen Dinh Duc et al[19] studied the mechanical and thermal stability of eccentrically stiffened functionally graded conical shell panels resting on elastic foundations in thermal environment by using classical shell theory and Lekhnitsky's smeared stiffeners technique. Galerkin method was used to solve the governing equations .

Galerkin method was adopted by R.D. Firouz-Abadi et al[20] for the analysis of free vibrations of moderately thick truncated conical shells filled with quiescent fluid. To obtain the governing equations of motion for the structure, Hamiltonian approach was employed. The final fluid equation is achieved by utilizing the Navier–Stokes equation with some assumptions like irrotational, compressible and inviscid etc.

Buckling and vibration analysis of a pressurized CNT reinforced functionally graded truncated conical shell under an axial compression using HDQ method was investigated by M. Mehri[21]. The equations of motion were established using Green-Lagrange type nonlinear kinematics within the framework of Novozhilov nonlinear shell theory and Donells shell theory. A semi-analytical solution on the basis of the trigonometric expansion through the circumferential direction was developed along with the harmonic differential quadrature (HDQ). To obtain the results, the stability equations were derived by applying Ritz method.

Saeed Kamarian et al[22] presented free vibration analysis of conical shells reinforced with agglomerated carbon nanotubes by using Eshelby-Mori-Tanaka approach based on an equivalent fiber assumption. The experimental data available from the literature were compared with the numerical results. The equations of motion were derived based on the first order shear deformation theory. The Generalized Differential Quadrature (GDQ) technique was originally implemented to solve the governing equations of the problem and to obtain the natural frequencies of the structures.



Using first order shear deformation theory Parviz Malekzadeh et al[23] investigated the thermal effect on free vibration of functionally graded truncated conical shell panels. Material properties were assumed to be graded in the thickness direction, and the initial equations of motion and the related boundary conditions were derived by using Hamilton's principle. The differential quadrature method was employed to discretize the equations of motion.

Winkler-Pasternak foundation effect on the frequency parameter of FGM truncated conical shells in the framework of shear deformation theory was studied by A. Deniz et al[24] using first order shear deformation theory[FSDT]. The combined effects of shear strains, material gradient and elastic foundations on the non-dimensional frequency parameters were studied. The eigenvalue problem was solved by using Galerkin method. Tanmoy Bandyopadhyay et al[25] investigated the transient response of delaminated composite conical shells due to multiple low velocity impacts in hygrothermal environment using higher order shear deformation theory and Midlin's theory. To compute the contact force and displacement arising from each impact on specific locations of delaminated conical shells, indentation laws were implemented. Final results were obtained by using Ritz method.

Reza Ansari and Jalal Torabi[26] gave the numerical study on the buckling and vibration of functionally graded carbon nanotube-reinforced composite conical shells under axial loading. The effective material properties of functionally graded composite conical shell were estimated based on the extended rule of mixture. The periodic differential operator is employed in circumferential direction and generalized differential quadrature (GDQ) method in axial direction. Numerical results were calculated by using Newton-Raphson method to discretize the equation of motion

The free vibration of sandwich truncated conical shells containing functionally graded layers within the shear deformation theory was investigated by A.H. Sofiyev and E. Osmancelebioglu[27]. Donnell kinematics assumptions were used to establish the governing equations. The problem was solved for piecewise and continuous through-the-thickness coatings stiffness variations. The closed-form solution was obtained for frequency parameters of truncated conical shell coated by functionally graded (FG) layers. All the equations were solved by using finite element method and Galerkin method to get the results.

A.H. Sofiyev[28] studied the stability analysis of shear deformable FGM sandwich conical shell under the axial load under different boundary conditions(freely supported). The basic equations of FGM sandwich conical shells were displayed based on the Donnell shell theory. Galerkin's method was utilized to convert the partial differential equations to algebraic equations.

Third order shear deformation theory(TSDT) was used by M. Nejati et al[29] to obtain the static and free vibration analysis of functionally graded conical shell reinforced by carbon nanotubes. The governing equations of motion for the rotating truncated composite conical shells were derived and solved numerically by means of the Generalized Differential Quadrature (GDQ) method. The GDQ numerical tool were used to solve complex problems without passing through any variational formulation, but solving directly the equations of motion in a strong form.

Thermal and mechanical stability of functionally graded carbon nanotubes(FGCNT) reinforced composite truncated conical shells surrounded by the elastic foundations was investigated by Nguyen



Dinh Duc et al[30] by using first order shear deformation theory. Classical shell theory was used to derive the equilibrium. The closed-form expressions were obtained by using Galerkin method.

Haar wavelet method was adopted by Qiyi Dai and Qingjie Cao[31] to present the parametric instability analysis of truncated conical shells. Using Bolotin's method, the first-order and second-order approximations of principal instability regions were determined. The present work was based on the first-approximation theory for classical thin shells. Dynamic instability behaviour of the shell was described by the partial differential equations. Numerical results were presented to bring out the influences of various parameters like static load factors, boundary conditions and shell geometrical characteristics on the domains of parametric instability of conical shells.

Francesco Tornabene et al[32] studied free vibration of composite conical panels reinforced with FG-CNTs. The kinematics of the problem were tackled with the first order shear deformation shell theory and the Donnell's theory. The virtual strain and kinetic energies expressions of the conical panels were obtained by using the Hamilton's principle. Ritz method was applied to find the results.

Layerwise first order shear deformation theory was adopted to analyze smart control and vibration of viscoelastic actuator-multiphase nanocomposite conical shells-sensor considering hygrothermal load by Reza Kolahchi et al[33]. Effective material properties were obtained by Halpin-Tsai model. The equations of motions were derived utilizing Hamilton's principle. The solution of the problem was carried out by differential quadrature method.

Multiphase nanocomposite viscoelastic laminated conical shells subjected to magneto-hygrothermal loads was presented by Reza Kolahchi et al[34]. The micromechanics and Halpin-Tsai equations in hierarchy were applied for calculating the effective material properties of the multiphase nanocomposite layers. The structural damping effects were considered based on Kelvin-Voigt theory. The surrounding medium is simulated by using visco-Pasternak model. The effects of various parameters such as structural damping, viscoelastic medium, magnetic field, number of layers, volume fraction of CNTs, temperature and moisture changes as well as boundary conditions on the dynamic instability region(DIR) of the structure were studied.

M.E. Fares et al[41] investigated the suppressing vibrational response of functionally graded truncated conical shells by active control and design optimization. To formulate the problem a shear deformation theory was used in different boundary conditions. Based on Lyapunov-Bellman theory, optimum values for the control forces and deflections were obtained for shells with simply supported or clamped edges.

First order shear deformation shell theory was used to present the buckling of functionally graded graphene reinforced conical shells under external pressure in thermal environment by Y. Kiani[42]. Donnell's kinematic assumptions and Von Karman geometrical non-linearity were used to establish the governing equations of the conical shell and the associated boundary conditions. Membrane analysis was employed to obtain the pre-buckling force of the shell. The linear stability equations were developed by using adjacent equilibrium criterion. These equations are discretized by means of the generalised differential quadrature method across the shell length and Fourier expansion through the circumferential direction.

A higher-order isoparametric superelement for free vibration analysis of functionally graded shells of revolution was investigated by Jalal Torabi and Reza Ansari[43]. The effective material properties of



functionally graded materials were varied continuously along the thickness direction of the shell . The governing equations which are based on the three-dimensional elasticity theory, were derived by using Hamilton's principle.

Free vibration analysis of functionally graded carbon nanotube reinforced composite truncated conical panels with general boundary conditions was studied by Qingshan Wang et al[44] by using Euler Bernoulli beam theory. The truncated conical panels were assumed to be graded through the thickness direction with different types of distributions reinforced by single-walled carbon nanotubes (SWCNTs).

III. CONCLUSION

In this paper author has tried to provide a thorough study of modal analysis of functionally graded conical shell.

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